An easy on-ramp to mastery-based grading

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April 29, 2023

Design Goals

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- High level of proficiency
- Many opportunities to get credit
- Easy to grade

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Department constraints

- Cumulative final worth at least 25%
- Mix of tests and homework matching other instructors in the same course

The Setup, Take 1

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Mastery Quizzes

- Take-home quiz each week
- Each topic is graded out of 2
- Best score on each topic counts
- Can get extra attempts by meeting with me
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Course Structure

- Two midterms: 15% each
- Final exam: 25%
- Online homework through WeBWorK platform: 15%
- Mastery Score: 30%

Calculus 1 Mastery Topics, Take 1

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1. Informal Continuity and Limits	12. Related Rates
2. Formal Limits	13. Critical Points and Extrema
3. Computing Limits	14. Relative Extrema
4. Trigonometric Limits	15. Curve Sketching
5. Infinite Limits	16. Optimization
6. Definition of a Derivative	17. Numerical Approximation
7. Computing Derivatives	18. Area and Riemann Sums
8. Trig and the Chain Rule	19. Integrals and the FTC
9. Linear Approximations and Tangent Lines	20. The Evaluation Theorem and Indefinite Integrals
10. Rates of Change	21. Integration by Substitution
11. Implicit Differentiation	22. Areas and Averages

Week 2 Quiz

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3. **Computing Limits** Compute:

   a. $\lim_{x \to 3} \dfrac{\sqrt{x+1} - 2}{x - 3} =$

   b. $\lim_{x \to 2} \dfrac{x^2 + x - 5}{3 - x} =$

   c. $\lim_{x \to 1} \dfrac{1}{x - 1} - \dfrac{1}{x^2 - x} =$

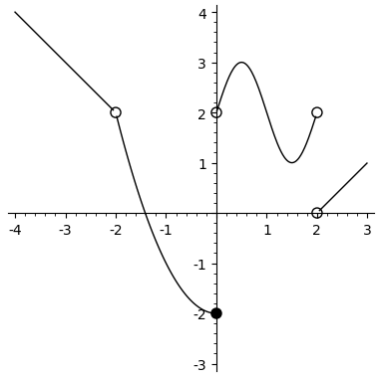
2. **Formal Limits**

   a. Write a formal ϵ-δ proof that $\lim_{x \to 2} 3x + 1 = 7$.

   b. Explicitly naming each limit law you use, compute $\lim_{x \to 2} 3 \cdot \dfrac{x^2 - 4}{x - 2}$.

1. **Informal Continuity/Limits**

   a. Give an approximate value for sin(.1), and explain how you got it.



   b. What is the domain of f? Where (if anywhere) is f discontinuous?

   c. What is $\lim_{x \to -2} f(x)$? What is $f(-2)$?

   d. What is $\lim_{x \to 0} f(x)$? What is $f(0)$?

Week 8 Quiz

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Table of Contents

New Rules

- At least three attempts on each question
- In future weeks you have to specifically request a topic
- May only answer three questions per week

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The Ugly

- 80 students per course
- Conceptual gerrymandering
- Needs non-mastery-quiz practice

The Rules, Take 2

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- Major topics: 4 tries, take best two
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Tests

- Organized along mastery topics
- Page for each major topic
- Question for each secondary topic

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- Homework-Only Topic
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  - Mean value theorem
- See-Only Topic
  - Challenging theoretical ideas
  - Interesting background asides
  - Proving derivative laws, proving the FTC

Calculus 1 Mastery Topics, Take 2

Major Topics	
1. Computing Limits	4. Extrema and Optimization
2. Computing Derivatives	5. Integration
3. Linear Approximation	6. Integral Applications
Secondary Topics	
1. Definition of a Limit	5. Related Rates
2. Squeeze Theorem	6. Curve Sketching
3. Definition of Derivative	7. Numeric Approximation
4. Rates of Change and Models	8. Riemann Sums

Sample Midterm

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M1: Compute the following limits.

(a) $\lim_{x\to 9} \frac{3-\sqrt{x}}{9-x}$

(b) $\lim_{x\to -\infty} \frac{3x^3+\sqrt[3]{x}}{\sqrt{9x^6+2x^2+1}+x}$

(c) $\lim_{x\to 1} \frac{\sin^2(x-1)}{(x-1)^2}$

(d) $\lim_{x\to 3} \frac{x-5}{(x-3)^2}$

S1: Suppose $f(x) = x^2 - 6x$, and we want an output of approximately -9. What input a should we aim for? Find a δ so that if our input is $a \pm \delta$ then our output will be -9 ± 2. Justify your answer.

M2: Compute the derivatives of the following functions.

(a) $f(x) = \sec\left(\frac{\sqrt{x^2+1}}{x+2}\right)$

(b) $g(x) = \sqrt[4]{\frac{x^3+\cos(x^2)}{\sin(x^3)+1}}$

S2: Show that $\lim_{x\to 0} x \sin\left(\frac{3}{x}\right) = 0.$

S3: **Directly from the definition of derivative,** compute the derivative of $f(x) = x^2 + \sqrt{x}$ at $a = 2$.

Calculus 1 Mastery Topics, Take 2.5

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1. Computing Limits	3. Extrema and Optimization
2. Computing Derivatives	4. Integration
Secondary Topics	
1. Definition of a Limit	6. Related Rates
2. Definition of Derivative	7. Curve Sketching
3. Linear Approximation	8. Numeric Approximation
4. Implicit Differentiation	9. Riemann Sums
5. Rates of Change and Models	10. Integral Applications

What do students think?

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Student Reviews

- His use of mastery-based grading is more fair and better representative of my learning progression in the course.
- With him providing multiple chances for weekly quizzes, I always thought he leaves space for students to learn and reflect questions that they got wrong.... [I] was surprised on how he trusts us to learn with less stress than others.
- The weekly MQ quizzes are an incredible way of actually learning the material without putting an extreme amount of stress and anxiety on students that other eng. classes do. GW should look to adopt this type of teaching for all of its engineering classes.

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- Better organized questions

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- Better organized questions
- "First week" problem for major topics
- In-class assessments
- Adapt for upper-division classes

Calculus 2 Mastery Topics, Take 1

1. Inverse Functions	2. Exponential and Logarithm
3. Derivatives of Exp and Log	4. Integrals with Exp and Log
5. Inverse Trig Functions	6. L'Hospital's rule
7. Integration by Parts	8. Trigonometric Integrals
9. Partial Fractions	10. Numeric Integration
11. Improper Integrals	12. Geometric Applications
13. Separable DEs	14. Sequences
15. Geometric and Telescoping Series	16. Divergence and Integral Tests
17. Comparison Test and Limit Comparison Test	18. Absolute and Conditional Convergence
19. Power Series	20. Power Series as Functions
21. Theory of Taylor Series	22. Computing Taylor Series
23. Applications of Taylor series	24. Parametrization

Calculus 2 Mastery Topics, Take 2

Major Topics	
1. Transcendental Functions	3. Series Convergence
2. Integration Techniques	4. Taylor Series
Secondary Topics	
1. Invertible Functions	6. Differential Equations
2. L'Hospital's Rule	7. Sequences and Series
3. Numeric Integration	8. Power Series
4. Taylor Series	9. Applications of Taylor Series
5. Geometric Applications	10. Parametrization

Multivariable Calculus Mastery Topics, Take 1

1. Lines and planes	2. Vector operations
3. Partial Derivatives and Linear Approximation	4. Gradient and directional derivatives
5. Multivariable optimization	6. Constrained optimization
7. Multivariable integrals	8. Integrals in other coordinate systems
9. Calculus of curves	10. Integral change of variables
11. Line integrals	12. Conservative Vector Fields
13. Surface integrals	14. Green's and Stokes's theorems
15. Divergence theorem	

Multivariable Calculus Mastery Topics, Take 2

Major Topics	
1. Vectors	2. Partial Derivatives
3. Optimization	4. Multiple Integrals
5. Line Integrals	6. Surface Integrals
Secondary Topics	
1. Lines and Planes	2. Vector Functions
3. Multivariable Functions	4. Integral Applications
5. Vector Fields	6. The Divergence Theorem

Linear Algebra Mastery Topics, Take 1

1. Systems of Linear Equations	2. Vector Equations and Spans
3. Linear Independence	4. Linear Transformations
5. Matrix Multiplication	6. Matrix Inverses
7. Subspaces	8. Basis and Dimension
9. Vector Spaces and Subspaces	10. Vector Space Linear Transformations
11. Bases and Coordinates	12. The Matrix of a Linear Transformation
13. Eigenvectors and Determinants	14. Characteristic Polynomials and Finding Eigensystems
15. Complex and Generalized Eigenvectors	16. Change of Basis
17. Similarity and Trace	18. Diagonalization
19. Dot Product and Projection	20. Inner Products
21. Orthogonal Decomposition	