Cutting Corners: You Miter Enjoy This Talk

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The DeWalt Manual

FIG. 15

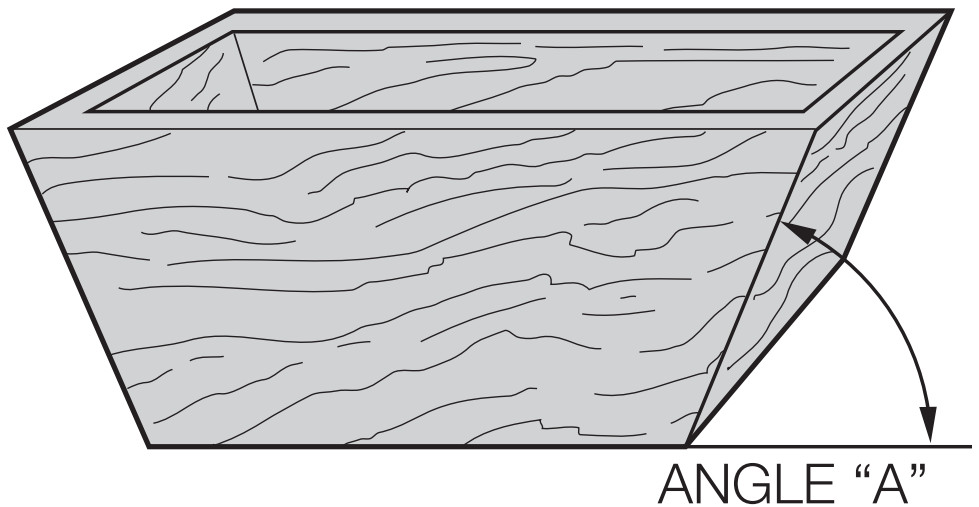
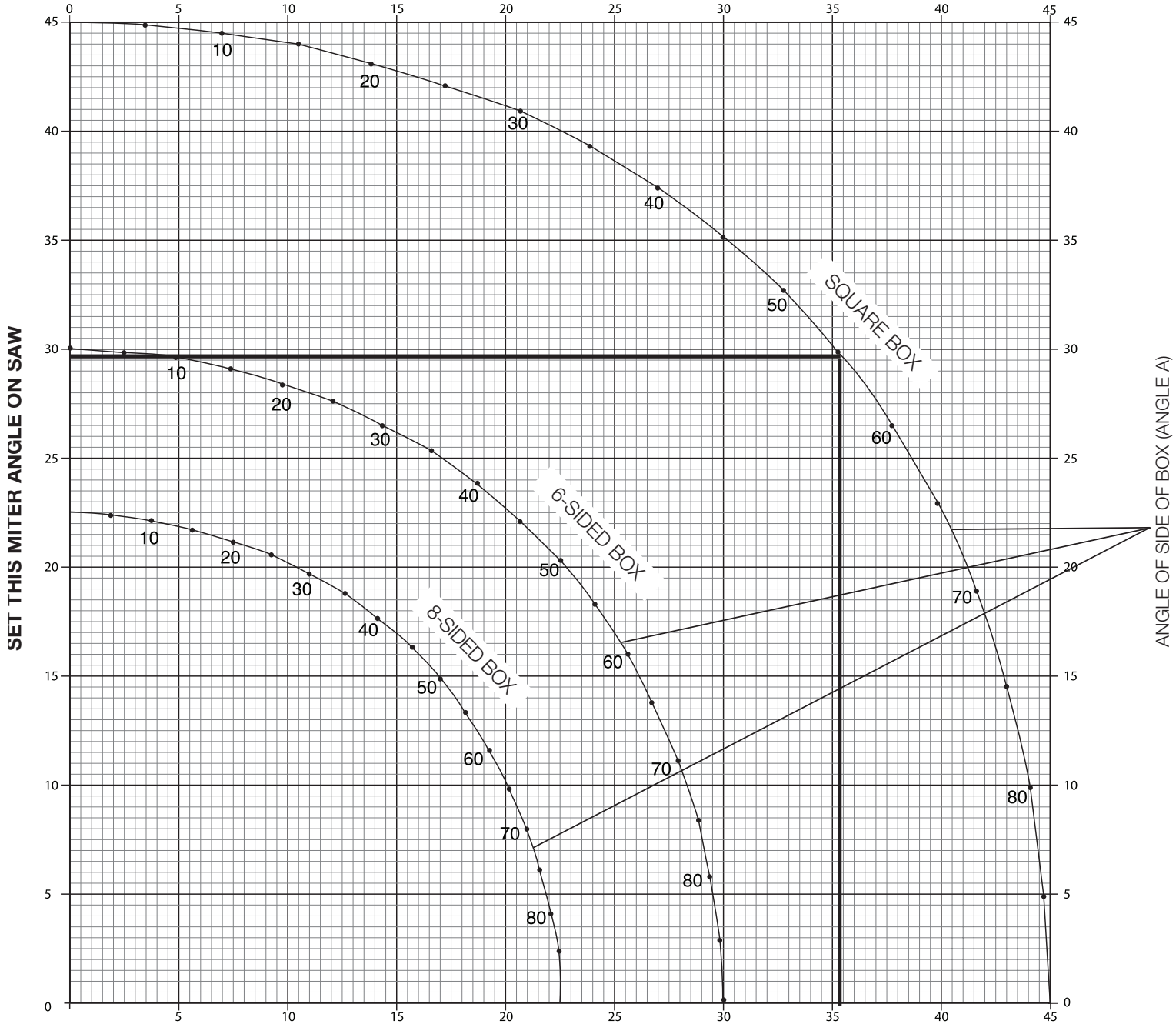


TABLE 1: COMPOUND MITER CUT

(POSITION WOOD WITH BROAD FLAT SIDE ON THE TABLE AND THE NARROW EDGE AGAINST THE FENCE)



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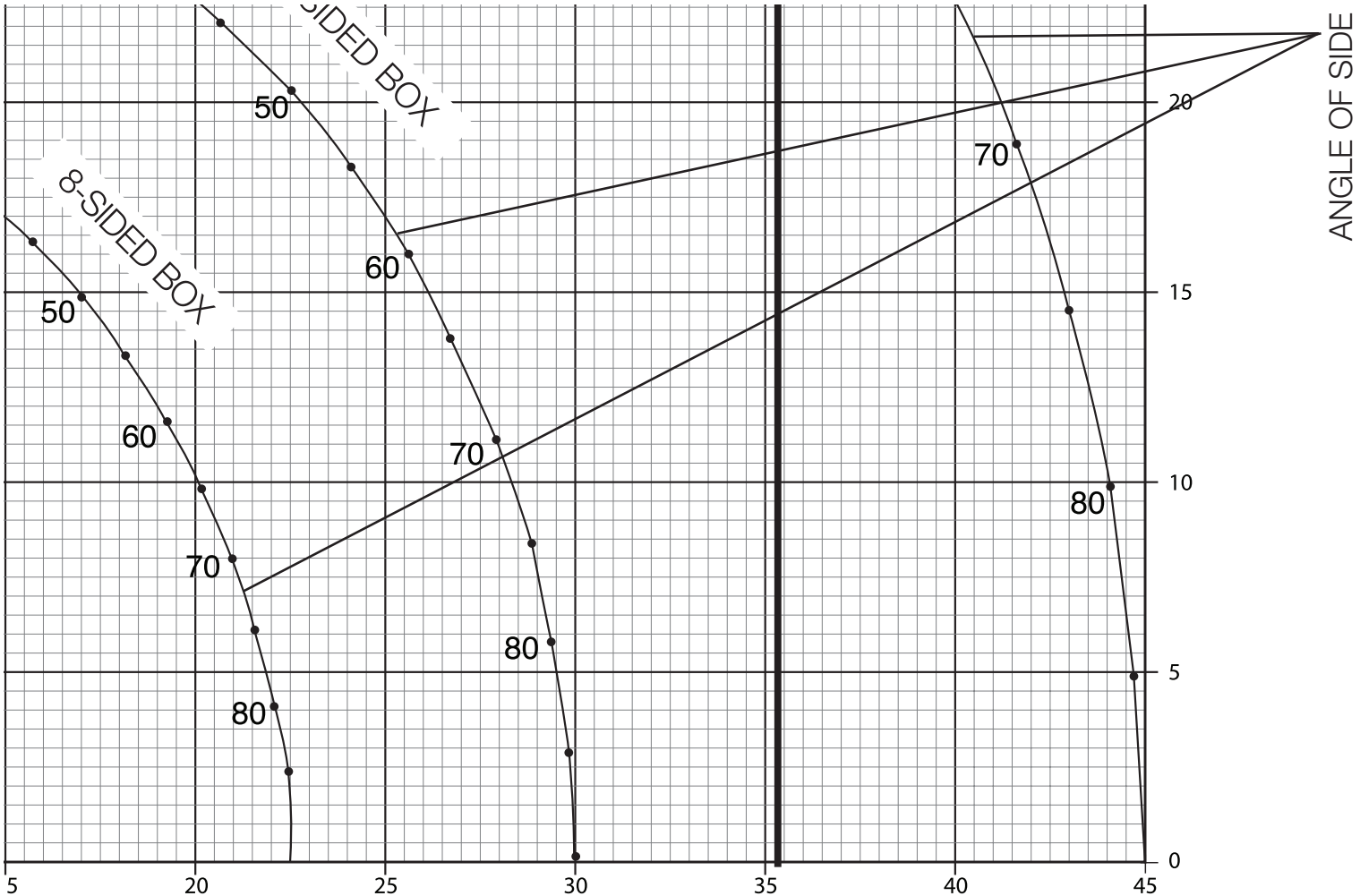

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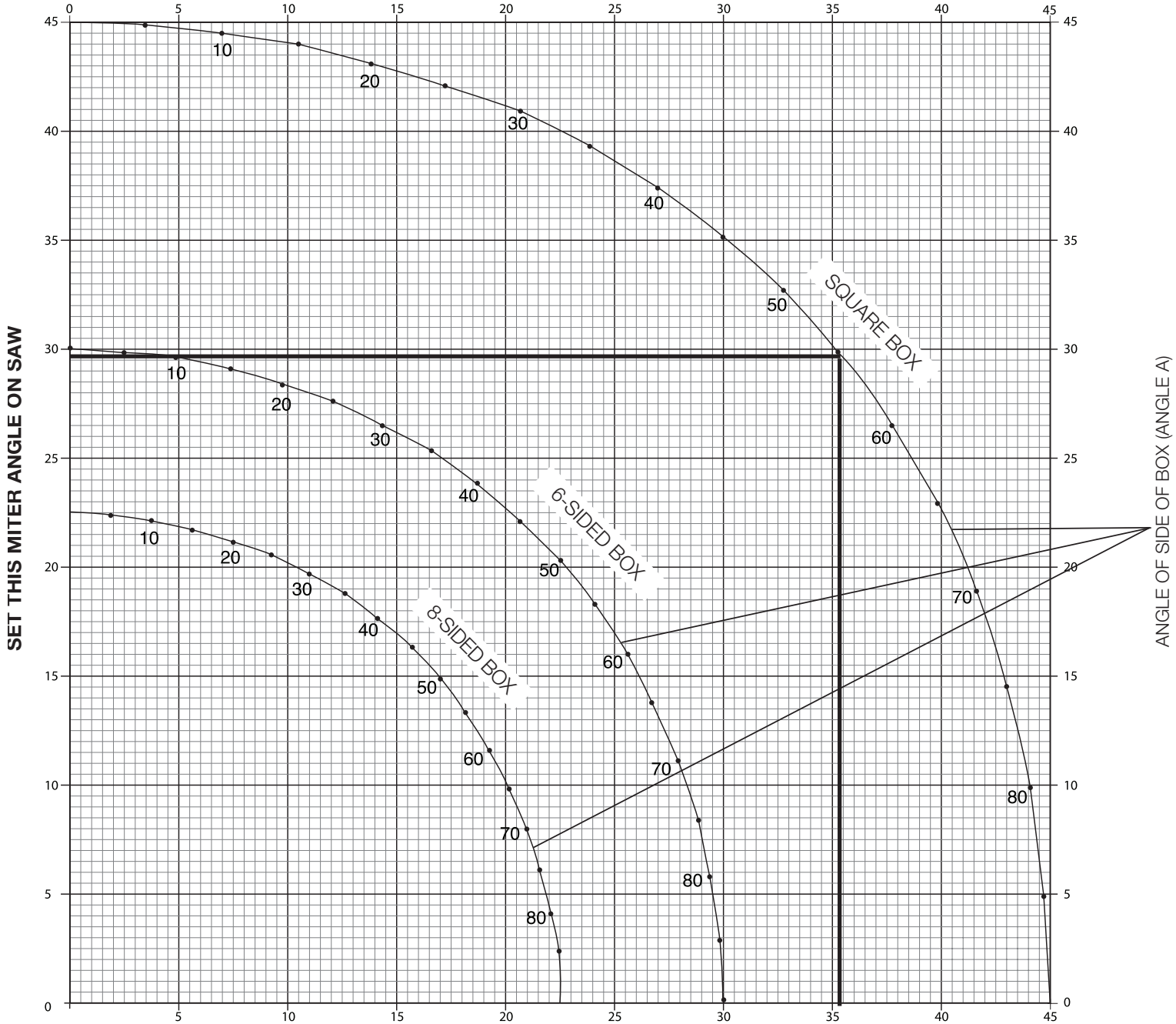

Orientation

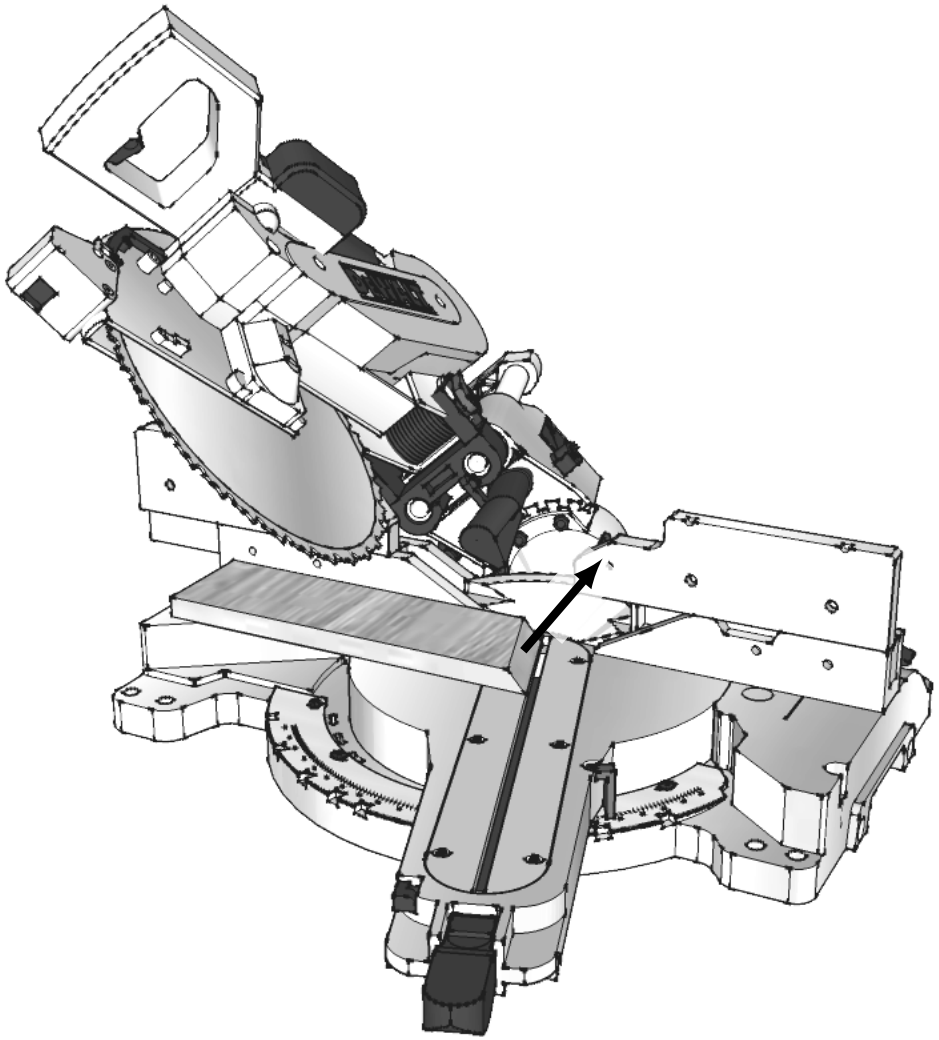

Image taken from 3D model: Inav, DW718 12 Double Bevel Sliding Compound Miter Saw. (2017). 3D Warehouse, Trimble Inc.

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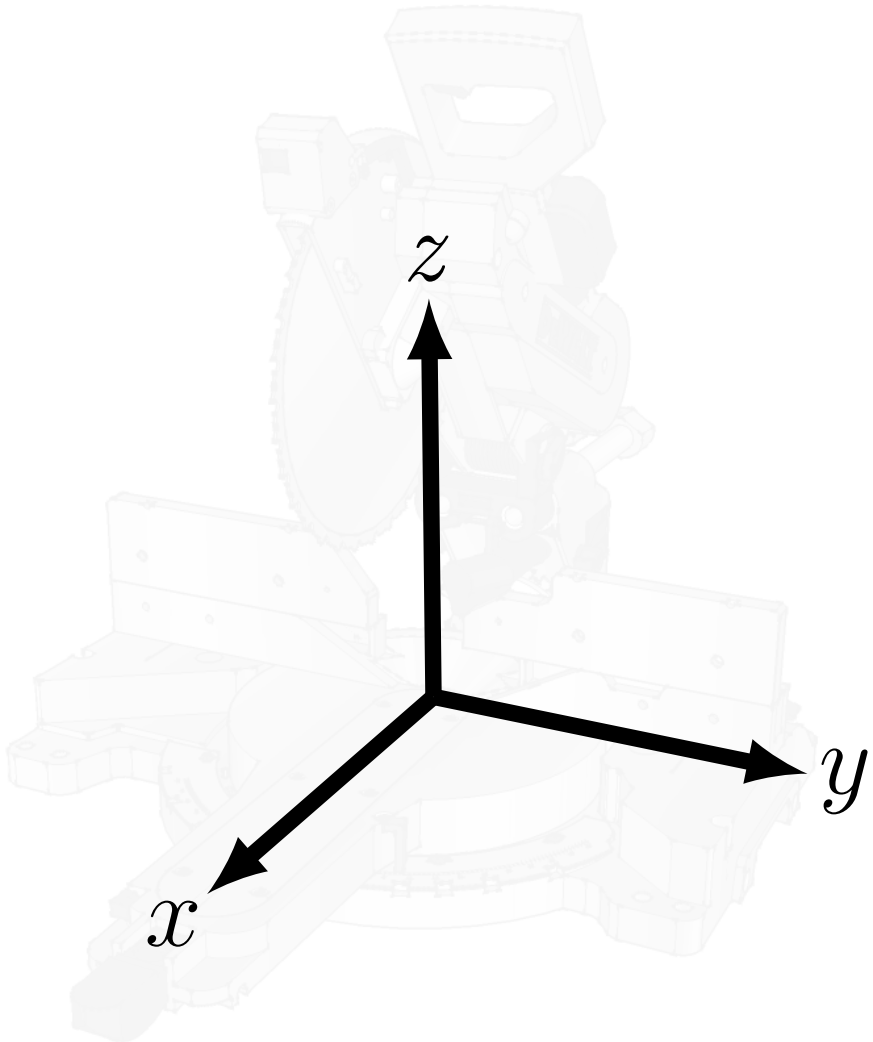

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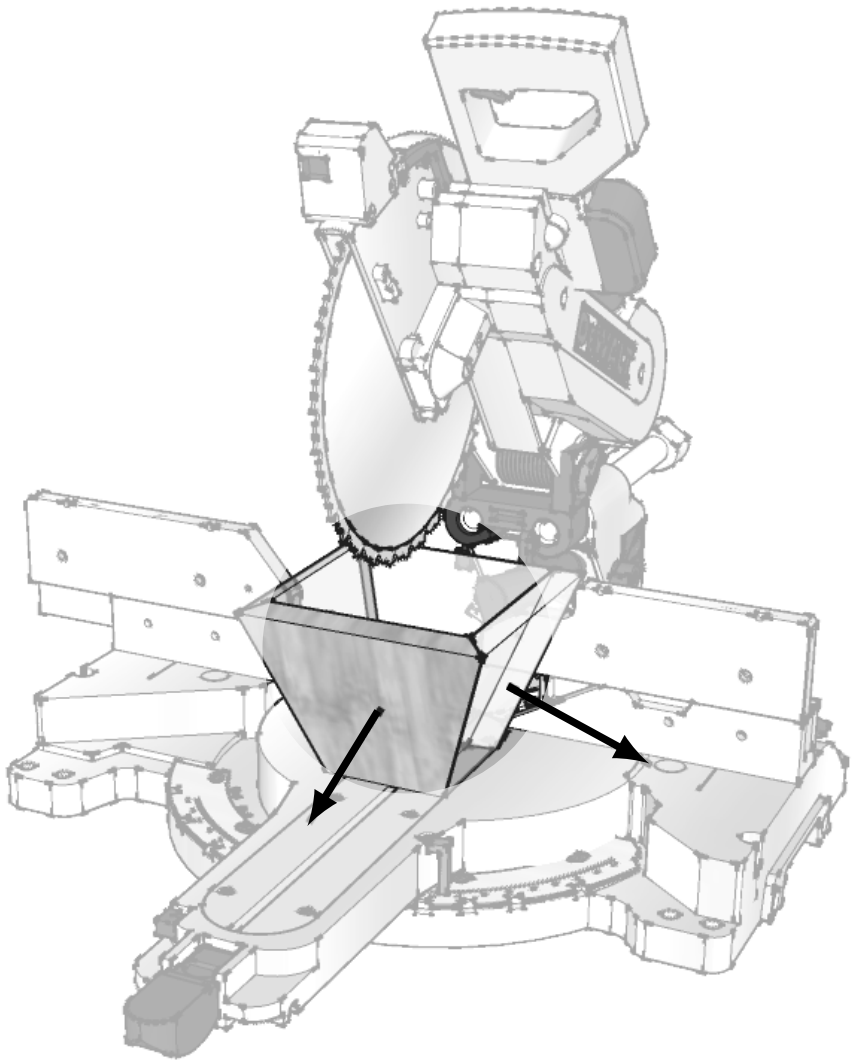

Rotation Matrices

$$M_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \qquad M_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

$$M_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Finding Bevel and Miter Angles

$\alpha =$ slope angle $\qquad \varphi =$ box exterior angle $(360^\circ/n)$

$b =$ bevel angle $\qquad m =$ miter angle

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$$m(\alpha,\varphi) = \tan^{-1}\big(\cos(\alpha)\tan(\varphi/2)\big)$$

Recreating the DeWalt Image

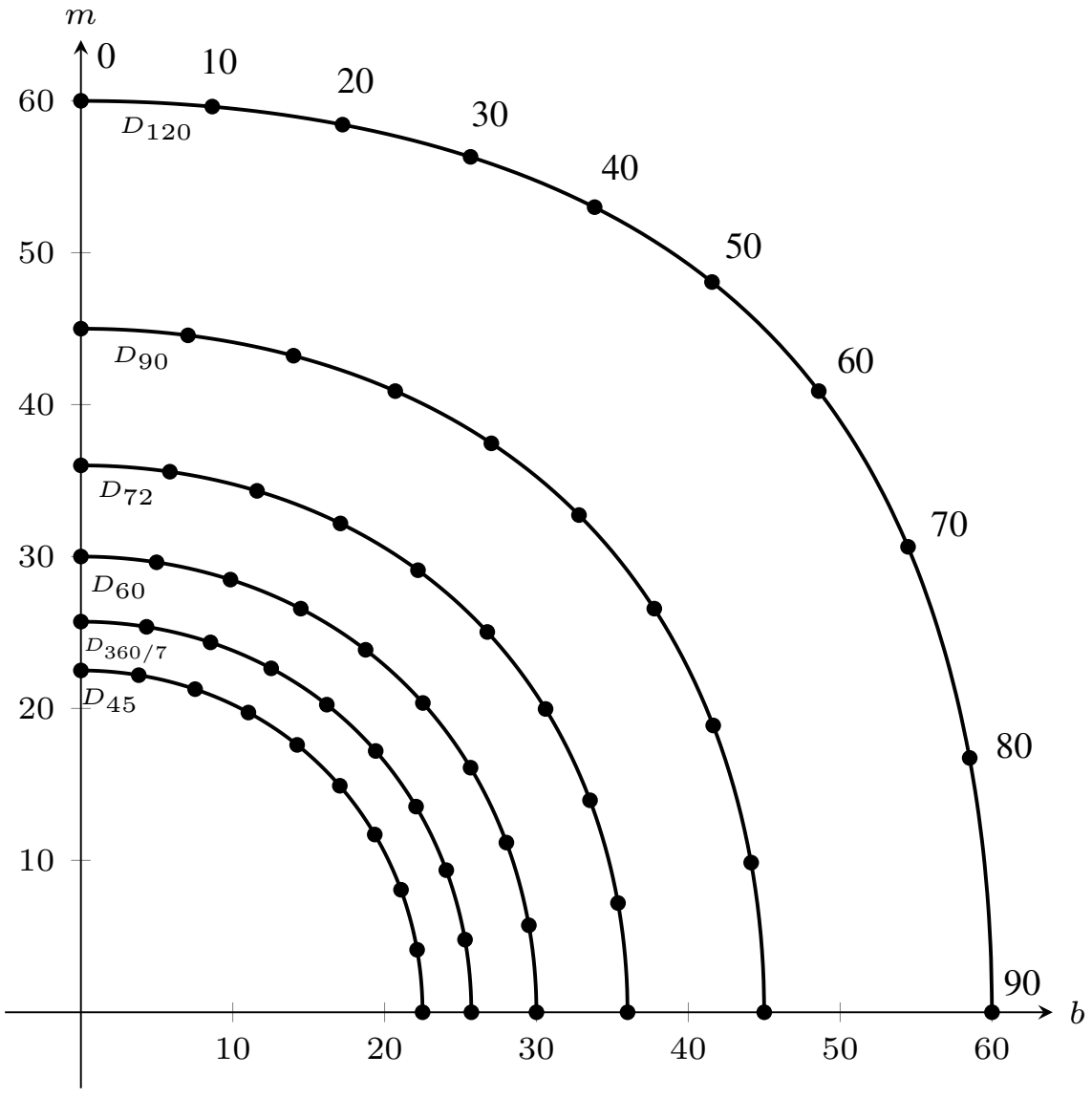
D-curves:

Fix $\varphi = \dfrac{360^\circ}{n}$.

$D_\varphi(\alpha)$ is parameterized by

$$x(\alpha) = b(\alpha, \varphi)$$

$$y(\alpha) = m(\alpha, \varphi)$$



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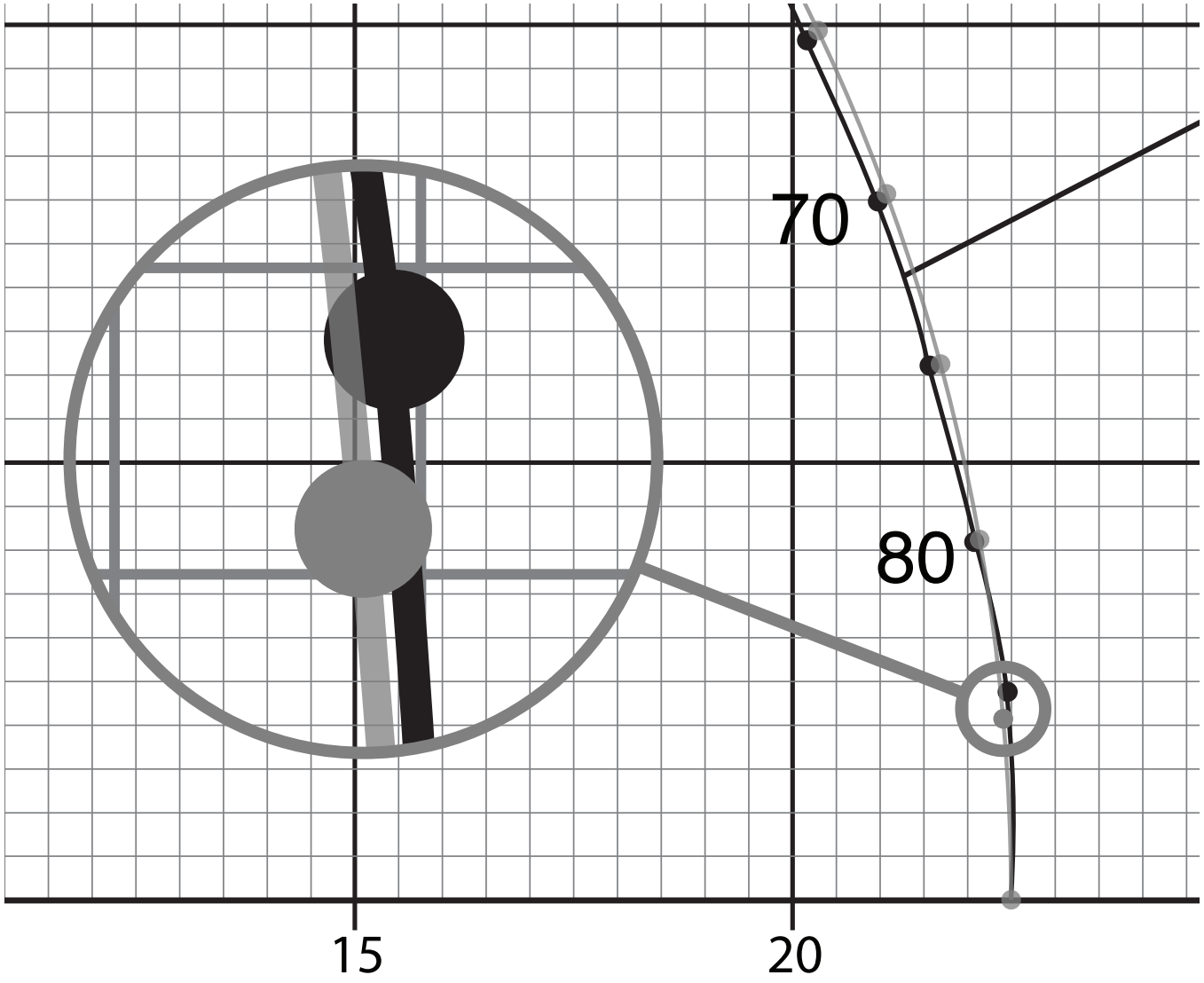
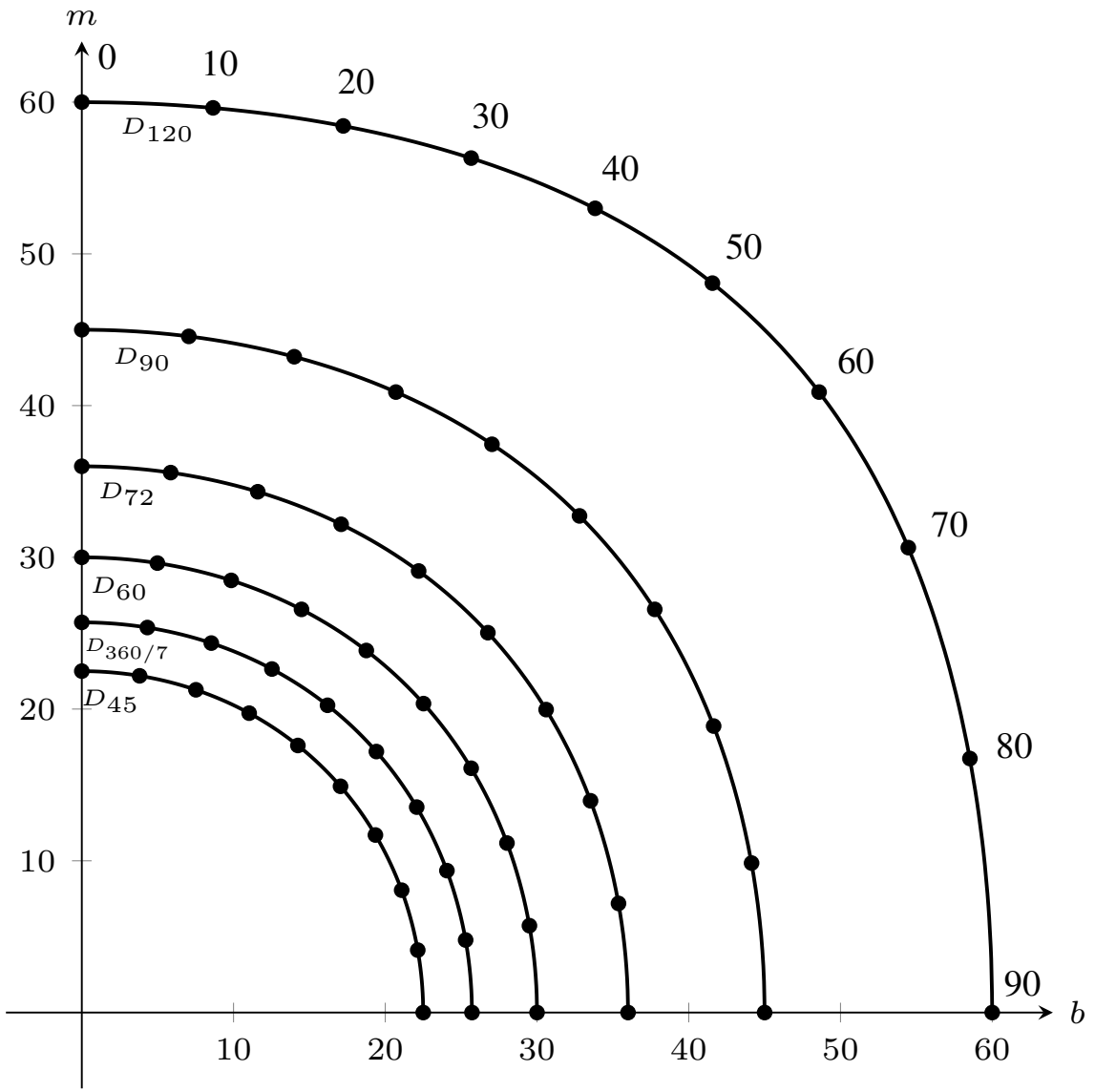


Properties of *D*-Curves – Symmetry



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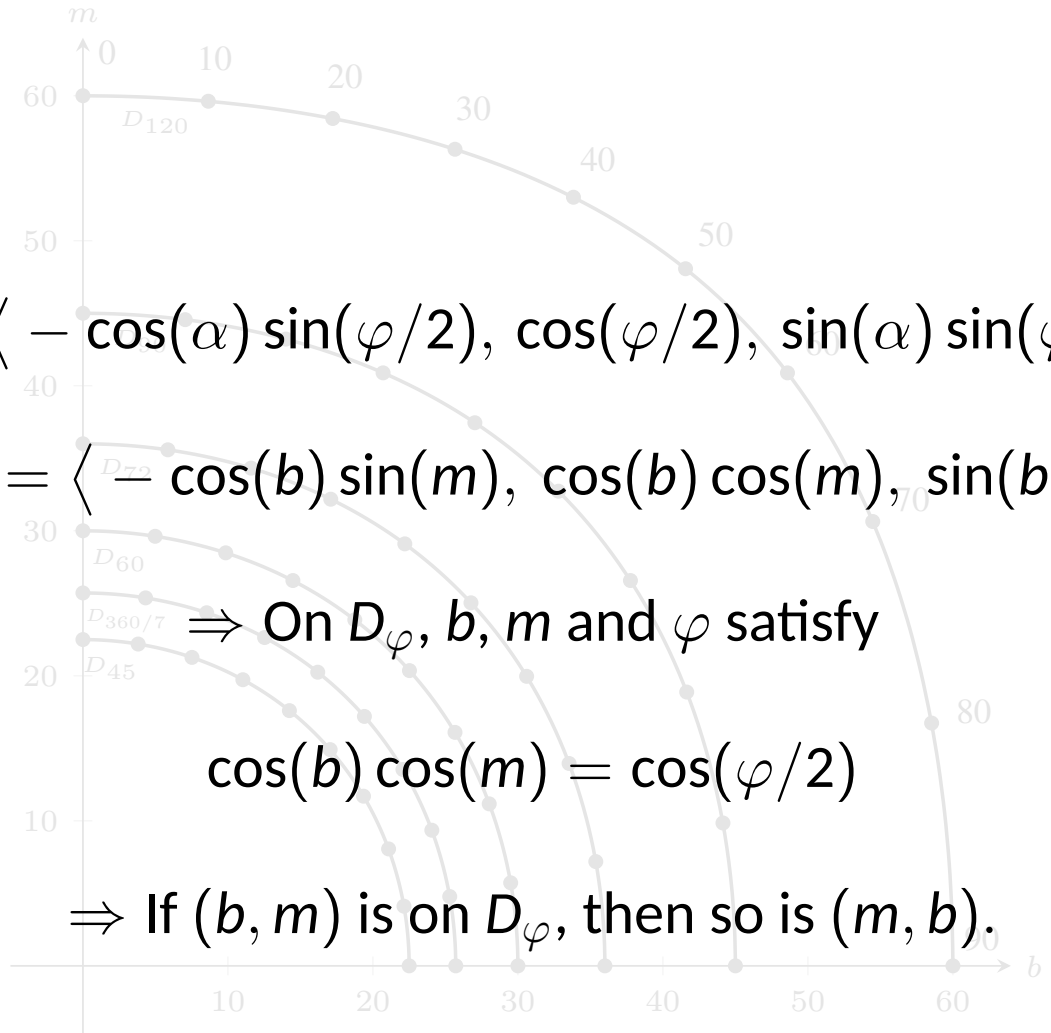


$$\vec{e} = \langle -\cos(\alpha)\sin(\varphi/2),\ \cos(\varphi/2),\ \sin(\alpha)\sin(\varphi/2)\rangle$$

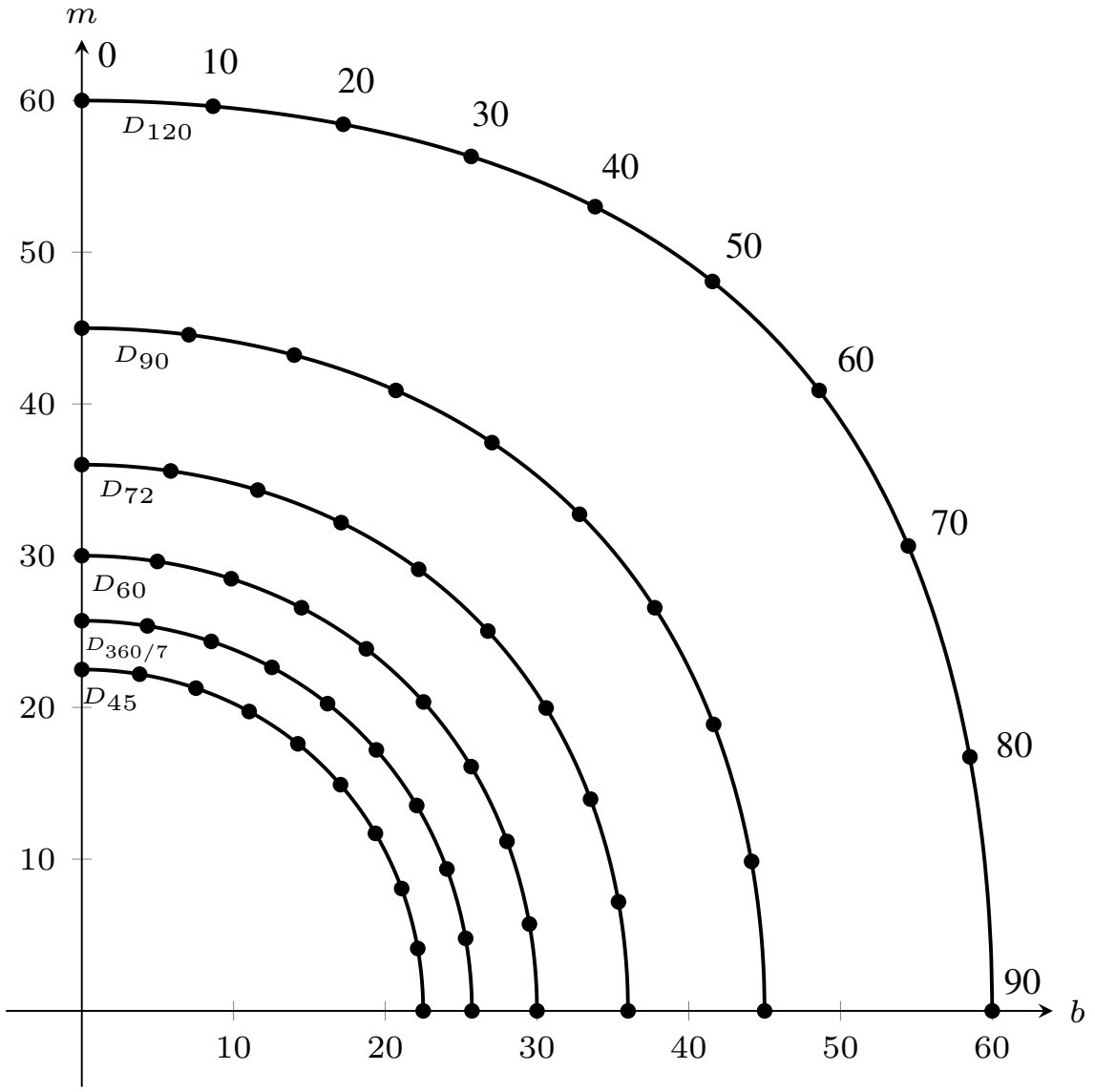
$$\vec{s} = \langle -\cos(b)\sin(m),\ \cos(b)\cos(m),\ \sin(b)\rangle$$

$\Rightarrow$ On D_φ, b, m and φ satisfy

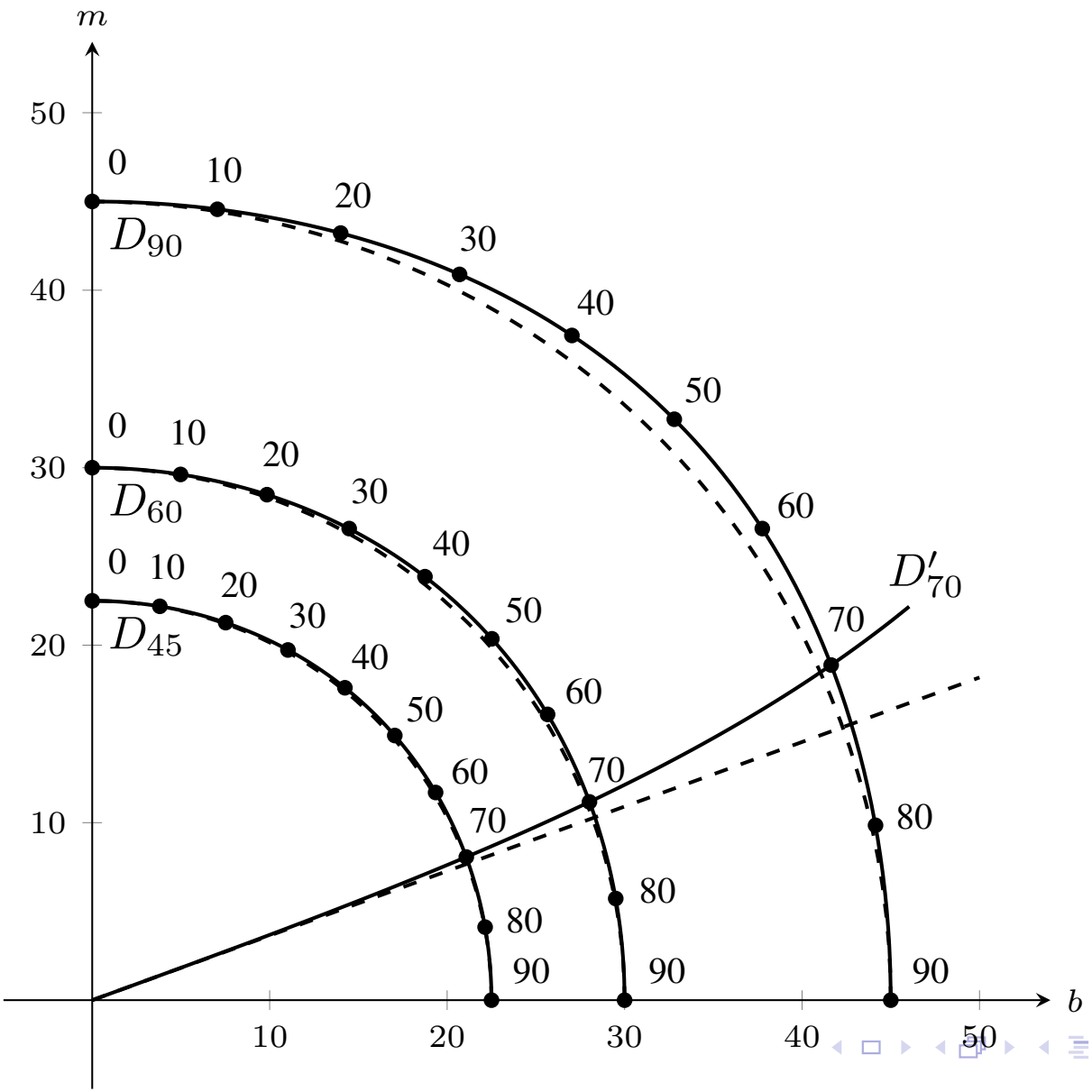
$$\cos(b)\cos(m) = \cos(\varphi/2)$$

$\Rightarrow$ If (b, m) is on D_φ, then so is (m, b).

Properties of *D*-Curves – Symmetry



Properties of *D*-Curves – Shape



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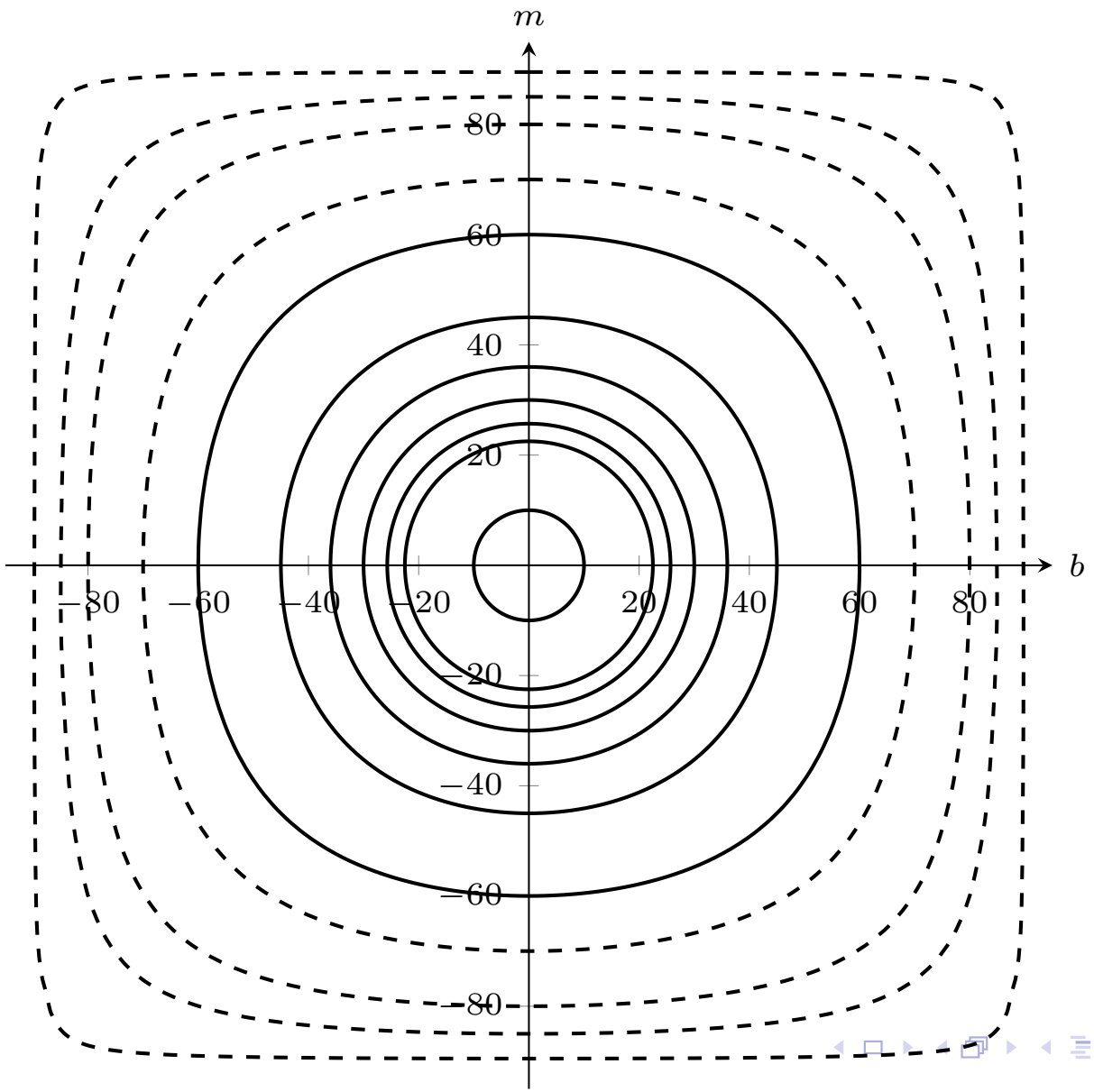
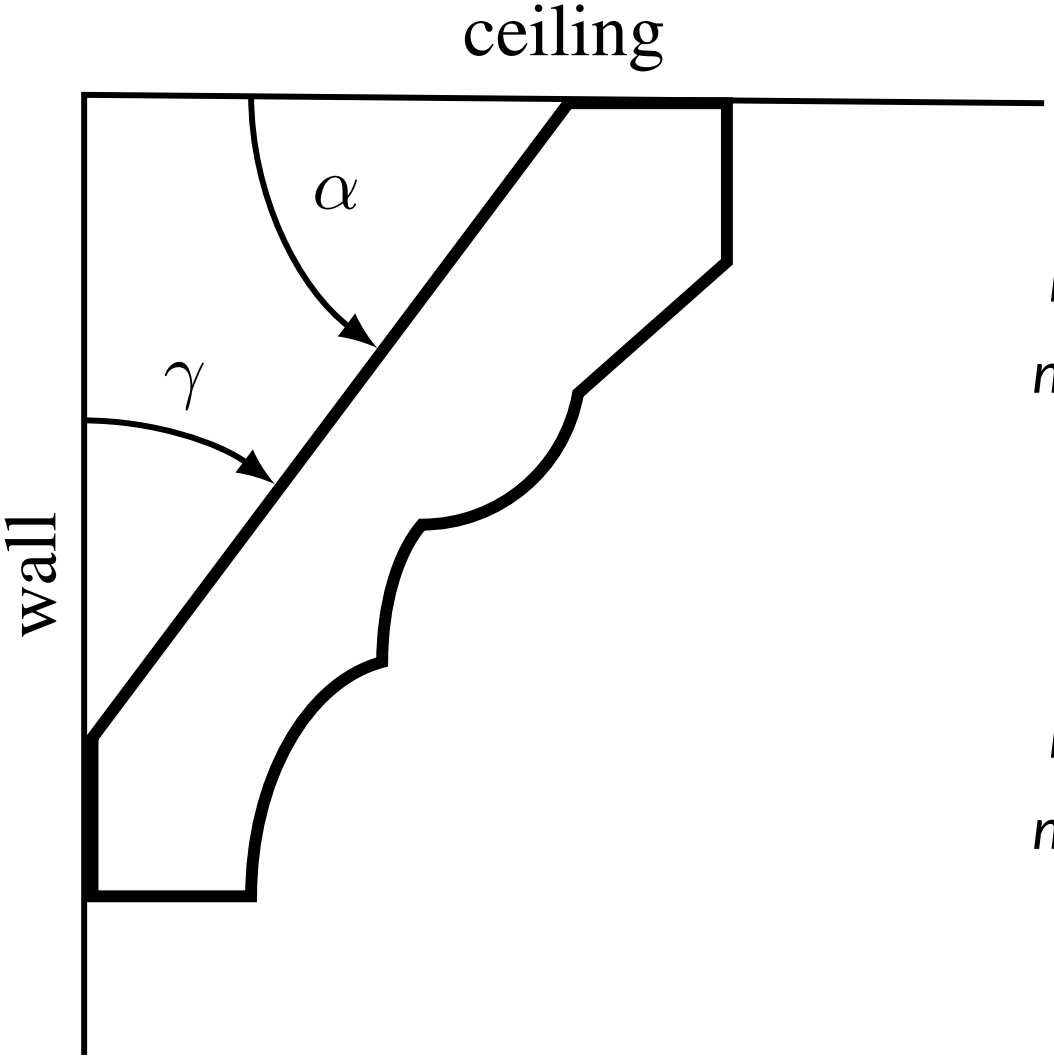


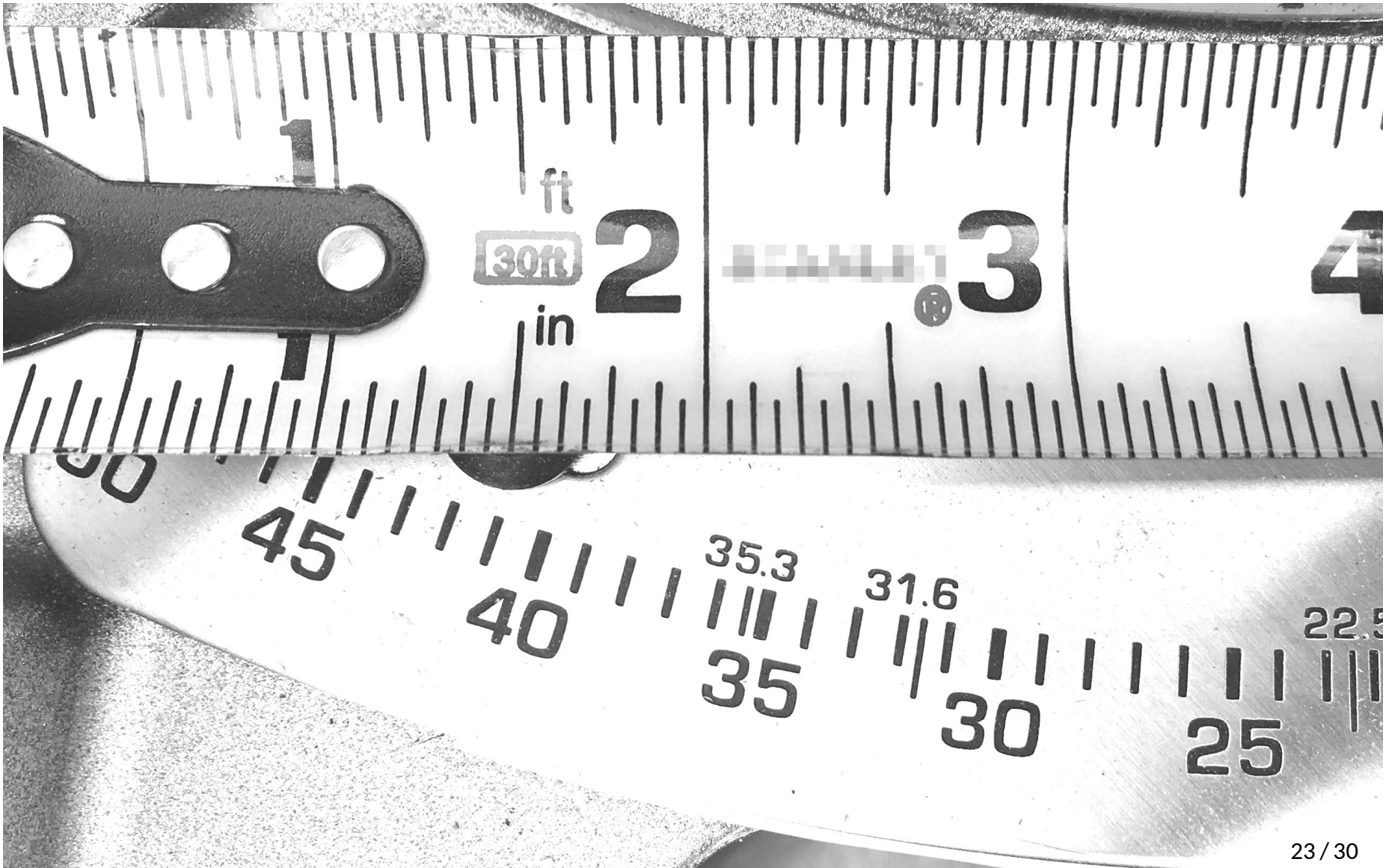
Application of *D*-Curves: Crown Molding



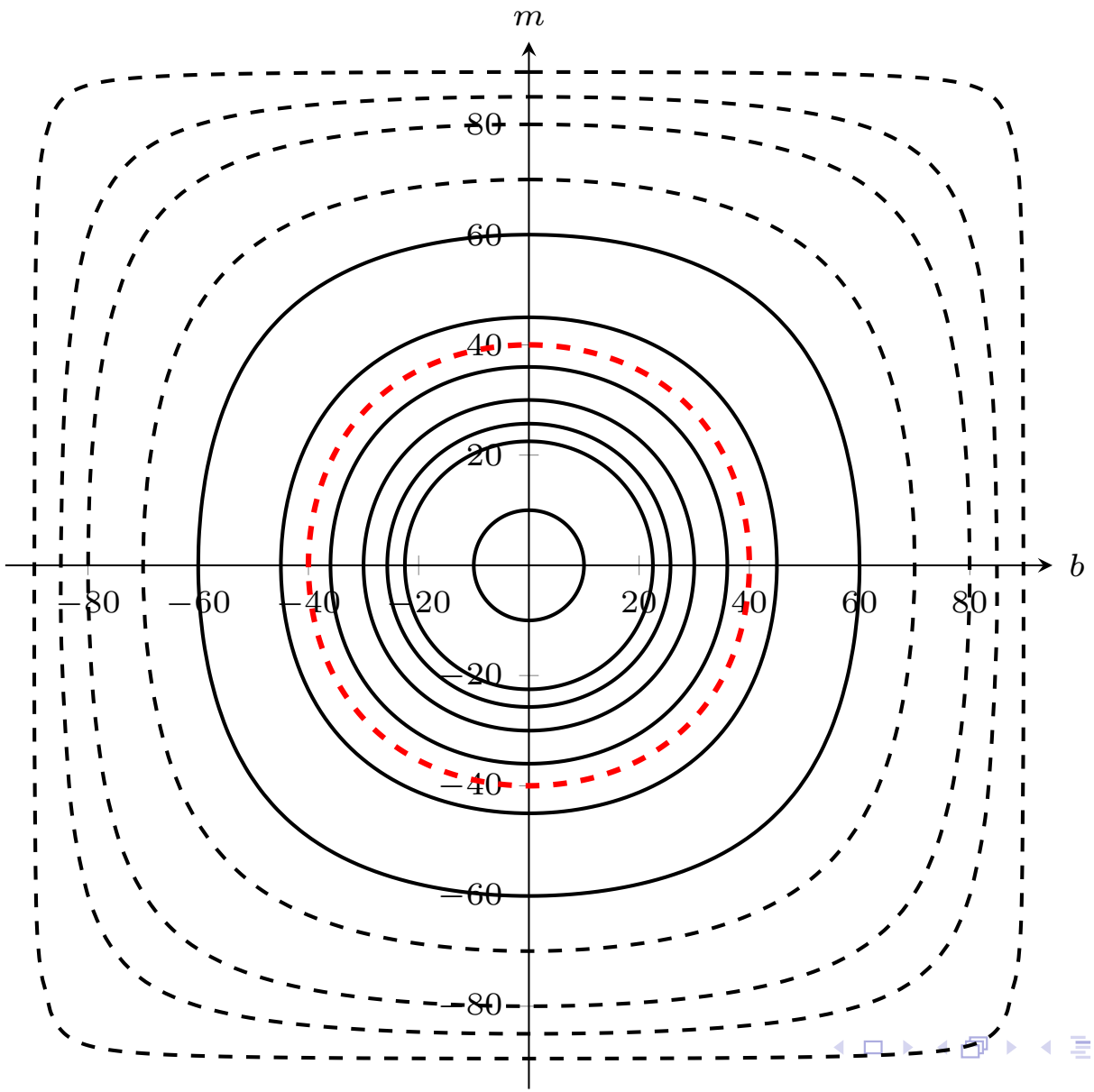
$$b(52^\circ, 90^\circ) = 33.86^\circ$$
$$m(52^\circ, 90^\circ) = 31.62^\circ$$

$$b(45^\circ, 90^\circ) = 30^\circ$$
$$m(45^\circ, 90^\circ) = 35.26^\circ$$

Application of *D*-Curves: Crown Molding



Properties of *D*-Curves – Fractional Sides?



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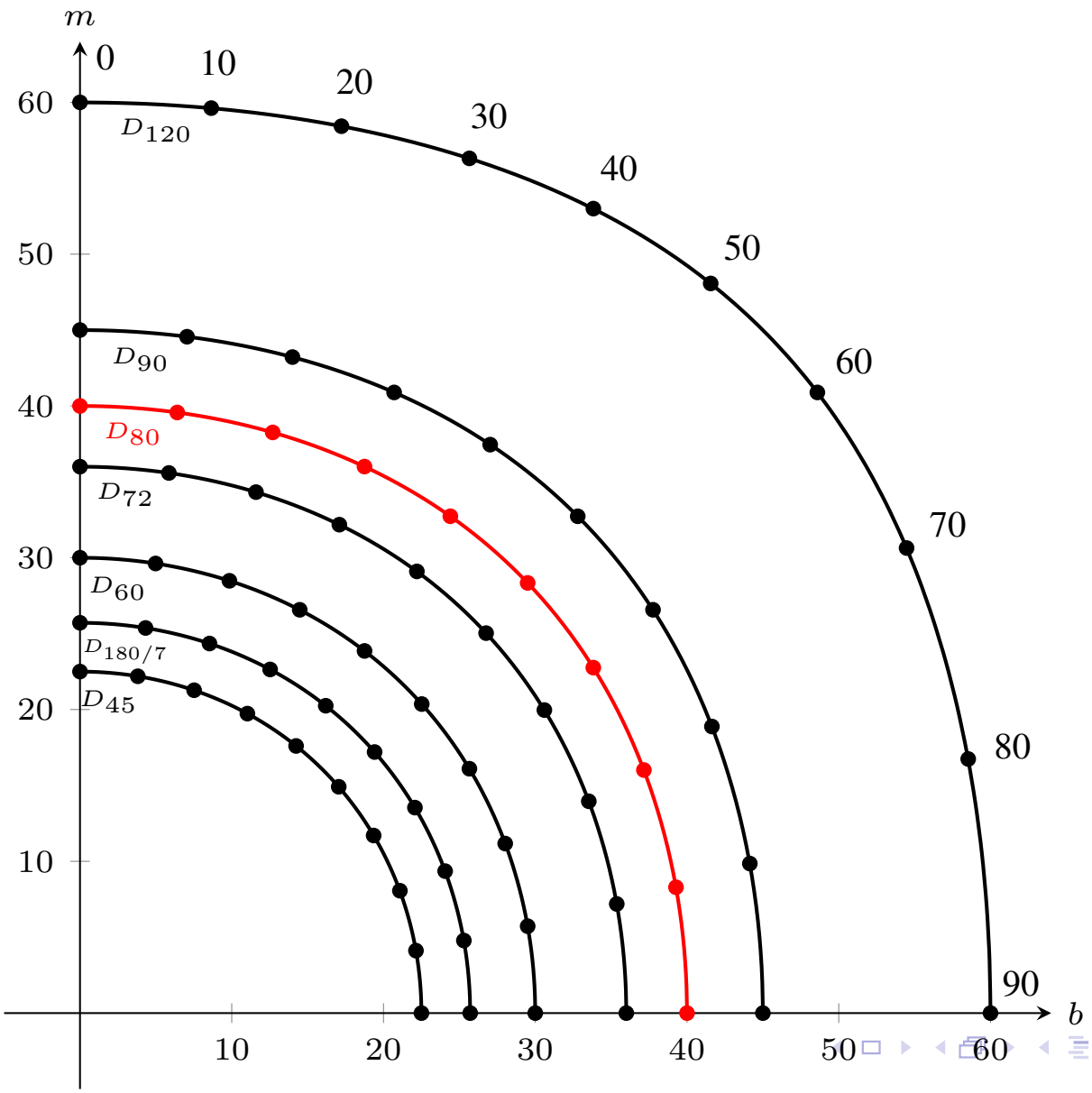
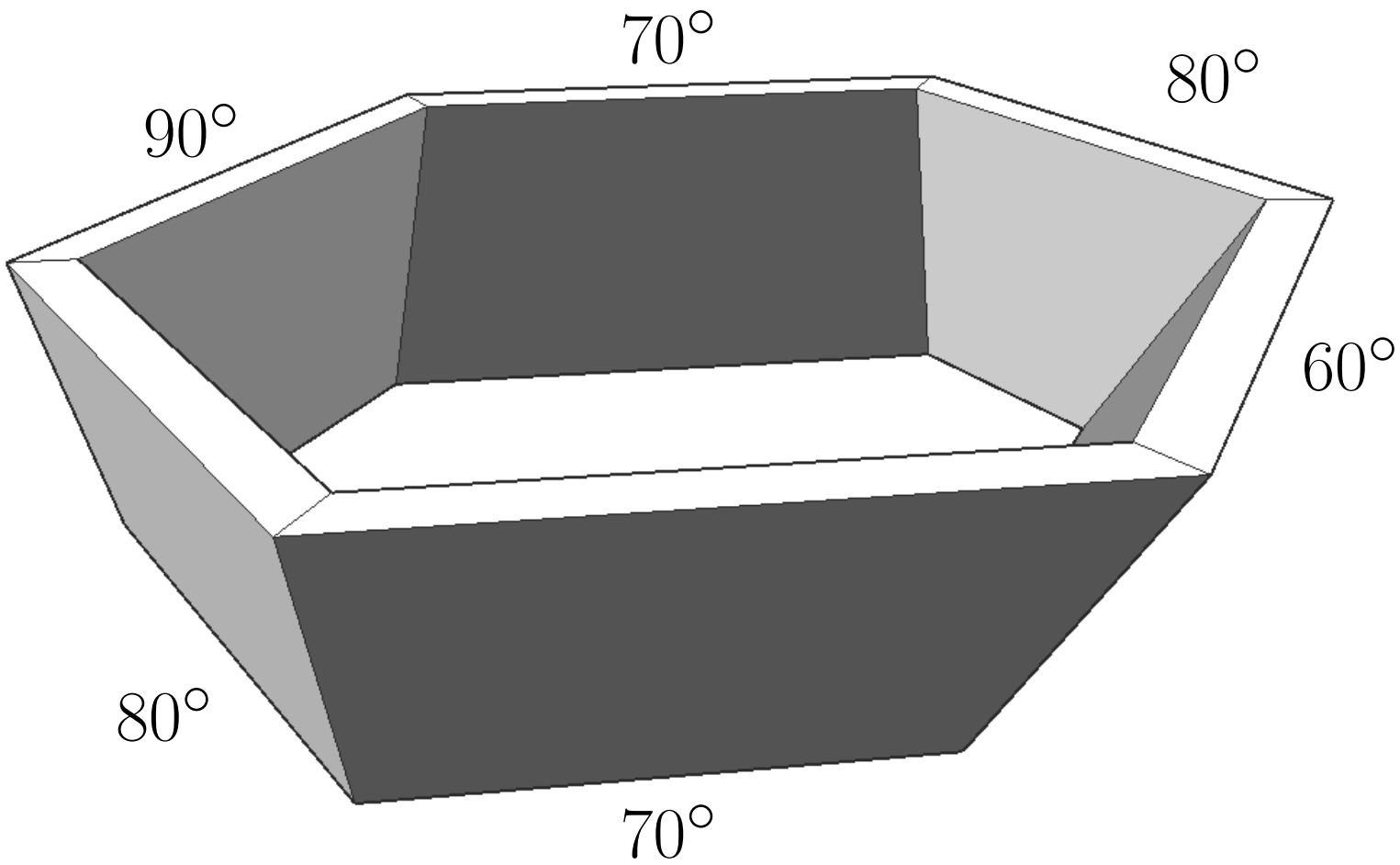


Boxes with Different Slope Angles



Boxes with Different Slope Angles

$\alpha =$ left slope angle $\qquad \beta =$ right slope angle $\qquad \varphi = 360^\circ / n$

Left edge of α-side:

$$b(\alpha, \beta, \varphi) = \sin^{-1}\left(\sqrt{\sin^2\left(\frac{\alpha - \beta}{2}\right) + \sin^2\left(\varphi/2\right)\sin(\alpha)\sin(\beta)}\right)$$

$$m(\alpha, \beta, \varphi) = \tan^{-1}\left(\frac{\sin(\alpha - \beta)}{\sin(\varphi)\sin(\beta)} + \cos(\alpha)\tan(\varphi/2)\right)$$

Right edge of β-side:

bevel: $-\, b(\beta, \alpha, \varphi)$

miter: $-\, m(\beta, \alpha, \varphi)$

Boxes with Different Slope Angles

Example:

Current Side Slope	Left Side Slope	Right Side Slope	BL	ML	BR	MR
70	90	80	29.3	-0.4	31.	-11.2
80	70	60	29.6	29.1	29.3	17.4
60	80	40	24.3	42.1	29.6	-6.4
40	60	50	21.2	10.2	24.3	-0.8
50	40	90	33.7	-20.4	21.2	34.3
90	50	70	31.	22.8	33.7	44.1

Questions *For* The Audience

- Where else do functions such as

$$b(\alpha, \varphi) = \sin^{-1}\left(\sin(\alpha)\sin(\varphi/2)\right)$$
$$m(\alpha, \varphi) = \tan^{-1}\left(\cos(\alpha)\tan(\varphi/2)\right)$$

  appear?
- Better way of writing $b(\alpha, \beta, \varphi)$ and $m(\alpha, \beta, \varphi)$?

Thanks!

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- Greg Hartman – hartmangn@vmi.edu