PreTeXt

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Mitchel T. Keller
Washington & Lee University

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Example 1.5.2. Suppose that $y = f(x)$ is a funct
known: $f(1) = 2.5$, $f(2) = 3.25$, and $f(3) = 3.625$. E
Solution

The intuitive approach to average the two estimate
fact the best possible estimate to $f'(2)$ when we ha
for f on opposite sides of the point of interest.

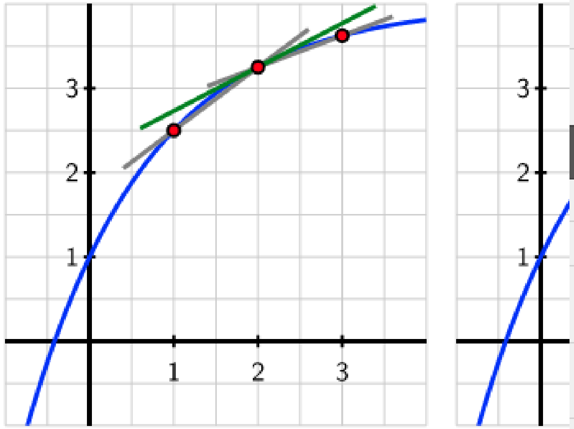

Figure 1.5.3. At left, the graph of $y = f(x)$ along wi
$(1, 2.5)$ and $(2, 3.25)$, the secant line through $(2, 3.2$
the tangent line. At right, the same graph along wit
$(1, 2.5)$ and $(3, 3.625)$, plus the tangent line.

To see why, we think about the diagram in Figure 1
function $y = f(x)$ that satisfies the data given in Ex
see the two secant lines with slopes that come fron
difference $\frac{f(1)-f(2)}{1-2} = 0.75$ and from the *forward diff*
how the first such line's slope over-estimates the sl
$(2, f(2))$, while the second line's slope underestima
however, we see the secant line whose slope is give

$f(3) - f(1) \quad 3.625 - 2.5 \quad 1.1$

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Example 1.5.2. Suppose that $y = f(x)$ is a function for which three values are known: $f(1) = 2.5$, $f(2) = 3.25$, and $f(3) = 3.625$. Estimate $f'(2)$.
Solution

We know that $f'(2) = \lim_{h \to 0} \frac{f(2+h)-f(2)}{h}$. But since we don't have a graph for $y = f(x)$ nor a formula for the function, we can neither sketch a tangent line nor evaluate the limit exactly. We can't even use smaller and smaller values of h to estimate the limit. Instead, we have just two choices: using $h = -1$ or $h = 1$, depending on which point we pair with $(2, 3.25)$.

So, one estimate is

$$f'(2) \approx \frac{f(1) - f(2)}{1 - 2} = \frac{2.5 - 3.25}{-1} = 0.75.$$

The other is

$$f'(2) \approx \frac{f(3) - f(2)}{3 - 2} = \frac{3.625 - 3.25}{1} = 0.375.$$

Since the first approximation looks only backward from the point $(2, 3.25)$ and the second approximation looks only forward from $(2, 3.25)$, it makes sense to average these two values in order to account for behavior on both sides of the point of interest. Doing so, we find that

$$f'(2) \approx \frac{0.75 + 0.375}{2} = 0.5625.$$

The intuitive approach to average the two estimates found in Example 1.5.2 is in fact the best possible estimate to $f'(2)$ when we have just two function values for f on opposite sides of the point of interest.

1.5.2 Toward more accurate derivative estimates

It is also helpful to recall, as we first experienced in Section 1.3, that when we want to estimate the value of $f'(x)$ at a given x, we can use the *difference quotient* $\frac{f(x+h)-f(x)}{h}$ with a relatively small value of h. In doing so, we should use both positive and negative values of h in order to make sure we account for the behavior of the function on both sides of the point of interest. To that end, we consider the following brief example to demonstrate the notion of a *central difference* and its role in estimating derivatives.

Example 1.5.2. Suppose that $y = f(x)$ is a function for which three values are known: $f(1) = 2.5$, $f(2) = 3.25$, and $f(3) = 3.625$. Estimate $f'(2)$.

Solution. We know that $f'(2) = \lim_{h\to 0} \frac{f(2+h)-f(2)}{h}$. But since we don't have a graph for $y = f(x)$ nor a formula for the function, we can neither sketch a tangent line nor evaluate the limit exactly. We can't even use smaller and smaller values of h to estimate the limit. Instead, we have just two choices: using $h = -1$ or $h = 1$, depending on which point we pair with $(2, 3.25)$.

So, one estimate is

$$f'(2) \approx \frac{f(1) - f(2)}{1 - 2} = \frac{2.5 - 3.25}{-1} = 0.75.$$

The other is

$$f'(2) \approx \frac{f(3) - f(2)}{3 - 2} = \frac{3.625 - 3.25}{1} = 0.375.$$

Since the first approximation looks only backward from the point $(2, 3.25)$ and the second approximation looks only forward from $(2, 3.25)$, it makes sense to average these two values in order to account for behavior on both sides of the point of interest. Doing so, we find that

$$f'(2) \approx \frac{0.75 + 0.375}{2} = 0.5625.$$

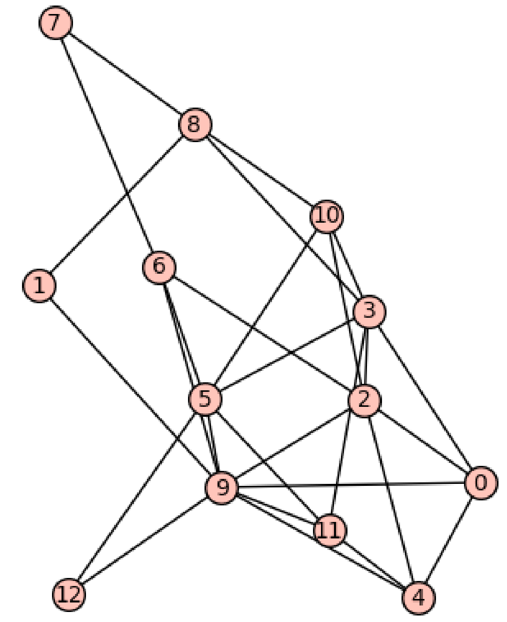
The intuitive approach to average the two estimates found in Example 1.5.2 is in fact the best possible estimate to $f'(2)$ when we have just two function values for f on opposite sides of the point of interest.

Run the code below. It will execute until it finds a graph $\mathbf{G}$ that is eulerian. The output that will be produced is a list of the degrees of the vertices of the graph $\mathbf{G}$ followed by a drawing of $\mathbf{G}$.

```
vertices = 13
edges = 28
g = graphs.RandomGNM(vertices,edges)
while (not g.is_eulerian() or not g.is_connected()):
    g = graphs.RandomGNM(vertices,edges)
print g.degree_sequence()
g.show()
```

Evaluate (Sage)

```
[8, 6, 6, 6, 4, 4, 4, 4, 4, 4, 2, 2, 2]
```



Chapter 15 The Sylow Theorems

We already know that the converse of Lagrange's Theorem is false. If G is a group of order m and n divides m, then G does not necessarily possess a subgroup of order n. For example, A_4 has order 12 but does not possess a subgroup of order 6. However, the Sylow Theorems do provide a partial converse for Lagrange's Theorem—in certain cases they guarantee us subgroups of specific orders. These theorems yield a powerful set of tools for the classification of all finite nonabelian groups.

Section 15.1 The Sylow Theorems

We will use what we have learned about group actions to prove the Sylow Theorems. Recall for a moment what it means for G to act on itself by conjugation and how conjugacy classes are distributed in the group according to the class equation, discussed in Chapter 14. A group G acts on itself by conjugation via the map $(g, x) \mapsto gxg^{-1}$. Let $x_1, \ldots, x_k$ be representatives from each of the distinct conjugacy classes of G that consist of more than one element. Then the class equation can be written as

$$|G| = |Z(G)| + [G : C(x_1)] + \cdots + [G : C(x_k)],$$

where $Z(G) = \{g \in G : gx = xg \text{ for all } x \in G\}$ is the center of G and $C(x_i) = \{g \in G : gx_i = x_i g\}$ is the centralizer subgroup of x_i.

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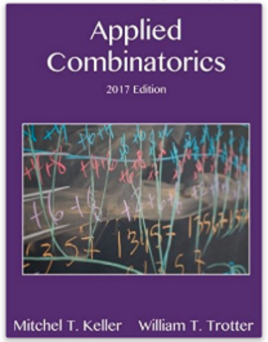

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```
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      Suppose that <m>y = f(x)</m> is a function for which three values are
known: <m>f(1) = 2.5</m>, <m>f(2) = 3.25</m>, and <m>f(3) = 3.625</m>.
Estimate <m>f'(2)</m>.
    </p>
  </statement>

  <solution>
    <p>
      We know that <m>f'(2) = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h}</m>. But since
we don't have a graph for <m>y = f(x)</m> nor a formula for the function, we can
neither sketch a tangent line nor evaluate the limit exactly. We can't even use smaller
and smaller values of <m>h</m> to estimate the limit. Instead, we have just two
choices: using <m>h = -1</m> or <m>h = 1</m>, depending on which point we
pair with <m>(2,3.25)</m>.
    </p>

    <p>
      So, one estimate is
      <me>
        f'(2) \approx \frac{f(1)-f(2)}{1-2} = \frac{2.5-3.25}{-1} = 0.75.
      </me>
    </p>

    <p>
      The other is
      <me>
        f'(2) \approx \frac{f(3)-f(2)}{3-2} = \frac{3.625-3.25}{1} = 0.375.
      </me
  </p>
```

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