WITTENGSTEIN WAS A SOCIAL CONSTRUCTIVIST

Ilhan M. Izmirli
George Mason University
iizmirl@gmu.edu

ABSTRACT

In this paper our main objective is to interpret the major concepts in Wittgenstein's philosophy of mathematics from a social constructivist point of view in an attempt to show that this philosophy is still very relevant in the way mathematics is being taught and practiced today.

1. A brief discussion of *radical constructivism*
2. A rudimentary analysis of the basic tenets of *social constructivism.* We observe that, the social constructivist epistemology of mathematics reinstates mathematics, and rightfully so, as "… a branch of knowledge which is indissolubly connected with other knowledge, through the web of language" (Ernest 1999), and portrays mathematical knowledge as a process that should be considered in conjunction with its historical origins and within a social context

ABSTRACT

3. The connections between social constructivism and Wittgenstein's philosophy of mathematics. Indeed, we argue that the apparent certainty and objectivity of mathematical knowledge, to paraphrase Ernest (Ernest 1998), rest on natural language.

Moreover, mathematical symbolism is a refinement and extension of written language: the rules of logic which permeate the use of natural language afford the foundation upon which the objectivity of mathematics rests. Mathematical truths arise from the definitional truths of natural language, and are acquired by social interaction. Mathematical certainty rests on socially accepted rules of discourse embedded in our *forms of life*, a concept introduced by Wittgenstein (Wittgenstein, 1956).

RADICAL CONSTRUCTIVISM

Constructivism, a movement associated with such figures as Immanuel Kant (1724-1804), John Dewey (1859-1952), and Jean Piaget (1896-1980), is an epistemological perspective which asserts that the concepts of science are mental constructs posited to elucidate our sensory encounters.

RADICAL CONSTRUCTIVISM

The fundamental tenets of constructivist epistemology are:

- Knowledge is a construct rather than a compilation of empirical data
- There is no single valid methodology in science, but rather a diversity of functional and effective methods
- One cannot focus on an ontological reality, but instead on a constructed reality. Indeed, search for ontological reality is entirely illogical, since to verify one has reached a definitive notion of Reality, one must already know what Reality consists of
- Knowledge and reality are products of their cultural context, that is, two independent cultures will likely form different observational methodologies

RADICAL CONSTRUCTIVISM

The term ***constructivist epistemology*** was first used by Jean Piaget in his famous 1967 article *Logique et Connaissance Scientifique* (Piaget 1967), but one can trace constructivist ideas back to

Heraclitus' adage *panta rhei* (everything flows),

Protagoras' claim that *man is the measure of all things*

The Socratic maxim "*I only know that I know nothing,"*

Pyrrhonian skeptics, who rejected the prospect of attaining truth either by sensory means or by reason, who, in fact, even considered the claim that nothing could be known to be dogmatic.

RADICAL CONSTRUCTIVISM

In 1970s, Ernst von Glasersfeld, who referred to the above type of constructivism as *trivial constructivism*, introduced the idea of *radical constructivism*, based on two premises:

- Knowledge is not passively received but actively built up by the cognizing subject
- The function of cognition is adaptive and serves the organization of the experiential world, not the discovery of ontological reality (Glasersfeld 1989, 162)

The term *radical* was used primarily to emphasize the fact that from an epistemological perspective, any constructivism had to be radical in order not to revert back into some form of realism.

RADICAL CONSTRUCTIVISM IN PHILOSOPHY OF MATHEMATICS: SOCIAL CONSTRUCTIVISM

In the modern philosophy of mathematics the major issue is the *absolutist* versus the *conceptual change* (*fallibilist*) dichotomy.

The absolutist philosophies, which date back to Plato, assert that mathematics is a compilation of absolute and certain knowledge,

The opposing *conceptual change* perspective contends that mathematics is a corrigible, fallible and transmuting social product (Putnam 2000).

SOCIAL CONSTRUCTIVISM

Absolutism makes two basic assumptions.

1. Mathematical knowledge is, in principle, separable from other human activities. It is discovered not invented.

2. Mathematical knowledge, logic, and the mathematical truths obtained through their applications are absolutely valid and eternally infallible. This second assumption can be written as

Certain established rules and axioms are true

- If p is a statement that is proven to be true at time t_0 then p is true at time $t_0 + t$, for any $t \geq 0$.
- Logical rules of inference preserve truth: If p is a true statement, and L is a logical rule of inference, then $L(p)$ is true.

SOCIAL CONSTRUCTIVISM

In sciences absolutist views, through the collective efforts of philosophers such as Karl Popper, Thomas Kuhn, Imre Lakatos, and Paul Feyerabend, have vanished.

However, among philosophers of mathematics the absolutist views nevertheless still prevail: mathematics is the epitome of certainty and mathematical truths are universal, and culture- and value-free. Its concepts are discovered, not invented.

SOCIAL CONSTRUCTIVISM

There are two major objections to mathematical absolutism.

1.As noted by Lakatos (1978), deductive logic, as the means of proof, cannot establish mathematical certainty for it inexorably leads to infinite regress - there is no way to elude the set of assumptions, however minimal, mathematical systems require. This even applies to definitions:

"What should we gain by a definition, as it can only lead us to other undefined terms?" (Wittgenstein 1965, 26)

-

SOCIAL CONSTRUCTIVISM

2. Even within an axiomatic system, mathematical theorems cannot be considered to be certain, for Gödel's Second Incompleteness Theorem demonstrates that consistency requires a larger set of assumptions than contained within any mathematical system.

SOCIAL CONSTRUCTIVISM

The *social constructivist* point of view (Paul Ernest) is rooted in the *radical constructivism* of Ernst von Glasersfeld. This point of view regards mathematics as a corrigible, and changing social construct, that is, as a cultural product fallible like any other form of knowledge. Presumed in this stance are two claims:

- The origins of mathematics are social or cultural
- The justification of mathematical knowledge rests on its quasi-empirical basis

SOCIAL CONSTRUCTIVISM

Hence, the absolutist philosophy of mathematics should be replaced by a philosophy of mathematics built upon principles of radical constructivism that, nevertheless, does not deny the existence of the physical and social worlds. This requires the incorporation of two extremely natural and undemanding assumptions, namely,

- The assumption of physical reality: There is an enduring physical world, as our common-sense tells us
- The assumption of social reality: Any discussion, including this one, presupposes the existence of the human race and language (Ernest 1999)

SOCIAL CONSTRUCTIVISM

The epistemological basis of social constructivism in mathematics:

- The personal theories which result from the organization of the experiential world must *fit* the constraints imposed by physical and social reality
- They achieve this by a cycle of theory-prediction-test-failure-accommodation-new theory
- This gives rise to socially agreed theories of the world and social patterns and rules of language use
- Mathematics is the theory of form and structure that arises within language.

THE THREE WITTGENSTEINS: *TRACTAUS LOGICO-PHILOSOPHICUS* vs. *THE MIDDLE PERIOD* vs. *PHILOSOPHICAL INVESTIGATIONS*

- Ludwig Josef Johann Wittgenstein (1889 – 1951) was an Austrian born British philosopher
- "perhaps the most perfect example … of genius as traditionally conceived, passionate, profound, intense, and dominating"
- An "arresting combination of monk, mystic, and mechanic,"
- He was a rather enigmatic, unfathomable character, at times deeply contemplative, at times utterly pugnacious, and almost always resplendent with inconsistencies and paradoxes.

THE THREE WITTGENSTEINS: *TRACTAUS LOGICO-PHILOSOPHICUS* vs. *THE MIDDLE PERIOD* vs. *PHILOSOPHICAL INVESTIGATIONS*

- Born into one of Europe's most opulent families, he gave away his entire inheritance.
- Three of his brothers committed suicide, and he constantly pondered it, as well.
- Though Jewish, he often expressed anti-Semitic feelings.

THE THREE WITTGENSTEINS: *TRACTAUS LOGICO-PHILOSOPHICUS* vs. *THE MIDDLE PERIOD* vs. *PHILOSOPHICAL INVESTIGATIONS*

A professor of philosophy at the University of Cambridge from 1939 until 1947, he left academia on several occasions - at times to travel to and to live in isolated areas for extended periods, at times to teach elementary school, and at times to serve as an ambulance driver - only to return each time.

He described philosophy as "the only work that gives me real satisfaction", yet, in his lifetime he published just one book of philosophy, the 75-page *Tractatus Logico-Philosophicus*, written in the trenches during World War I.

THE THREE WITTGENSTEINS: *TRACTAUS LOGICO-PHILOSOPHICUS* vs. *THE MIDDLE PERIOD* vs. *PHILOSOPHICAL INVESTIGATIONS*

His *Philosophical Investigations* (ranked as one of the most significant philosophical tour de forces of twentieth century philosophy)was posthumously published in 1953.

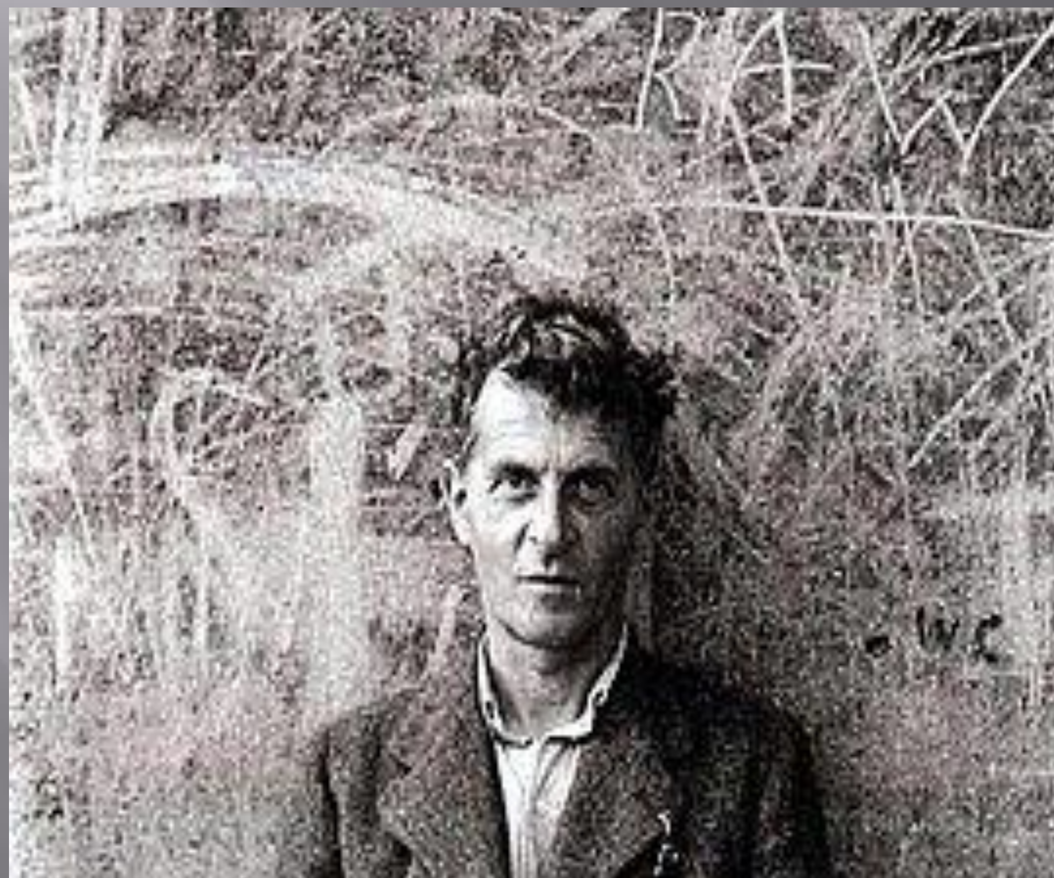
Although Wittgenstein worked primarily in logic and the philosophy of language, his contributions to the philosophy of mathematics were quite substantial and noteworthy . Indeed, Wittgenstein, who devoted the majority of his writings from 1929 to 1944 to mathematics, himself said that his

"… chief contribution has been in the philosophy of mathematics" (Monk 1990, 40).

THE THREE WITTGENSTEINS: *TRACTAUS LOGICO-PHILOSOPHICUS* vs. *THE MIDDLE PERIOD* vs. *PHILOSOPHICAL INVESTIGATIONS*

It is customary to distinguish three periods in Wittgenstein's philosophy of mathematics: The early period characterized by the concise treatise *Tractatus Logico-Philosophicus,* the middle period exemplified by such works as *Philosophical Remarks*, *Philosophical Grammar*, and *Remarks on the Foundations of Mathematics,* and the late period embodied by *Philosophical Investigations*.

THE THREE WITTGENSTEINS: *TRACTAUS LOGICO-PHILOSOPHICUS* vs. *THE MIDDLE PERIOD* vs. *PHILOSOPHICAL INVESTIGATIONS*



TRACTAUS LOGICO-PHILOSOPHICUS

The aim of the *Tractatus* was to reveal the relationship between language and the world, that is to say, to identify the association between language and reality and to define the limits of science.

Bertrand Russell who as a logical atomist pioneered the rigorous use of the techniques of logic to elucidate the relationship between language and the world.

According to logical atomists all words stood for objects. So, for instance, for a logical atomist the word "computer" stands for the object computer. But then what object does "iron man" signify?

TRACTAUS LOGICO-PHILOSOPHICUS

Russell’s famous example, is the phrase “The King of France is bald.” This is an utterly coherent construction but what does “the King of France” stand for? Russell construed that to think of “the King of France” behaving like a name was causing us to be confused by language.

He posited that this sentence, in fact, was formed of three logical statements:

- There is a King of France.
- There is only one King of France.
- Whatever is King of France is bald.

TRACTAUS LOGICO-PHILOSOPHICUS

The early Wittgenstein, like Russell, believed that everyday language obscured its underlying logical structure. He argued that language had a core logical structure, a structure that established the limits of what can be said meaningfully: In fact, he wrote in the preface:

“The book will, therefore, draw a limit to thinking, or rather—not to thinking, but to the expression of thoughts; for, in order to draw a limit to thinking we should have to be able to think both sides of this limit (we should therefore have to be able to think what cannot be thought).“

TRACTAUS LOGICO-PHILOSOPHICUS

Much of philosophy, Wittgenstein claimed, involved attempts to verbalize what in fact could not be verbalized, and that by implication should be unthinkable:

"What can we say at all can be said clearly. Anything beyond that—religion, ethics, aesthetics, the mystical—cannot be discussed. They are not in themselves nonsensical, but any statement about them must be."

TRACTAUS LOGICO-PHILOSOPHICUS

Tractatus
Logico-Philosophicus

By
LUDWIG WITTGENSTEIN

With an Introduction by
BERTRAND RUSSELL, F.R.S.

NEW YORK
HARCOURT, BRACE & COMPANY, INC.
LONDON: KEGAN PAUL, TRENCH, TRUBNER & CO., LTD.
1922

TRACTAUS LOGICO-PHILOSOPHICUS

Tractatus was devoted to explaining what a meaningful proposition was - what was asserted when a sentence was used meaningfully. It comprised propositions numbered from one to seven, with various sub-levels denoted 1, 1.1, 1.11, …

TRACTAUS LOGICO-PHILOSOPHICUS

1. *Die Welt ist alles, was der Fall ist*. The world is all that is the case.
2. *Was der Fall ist, die Tatsache, ist das Bestehen von Sachverhalten*. What is the case—a fact—is the existence of states of affairs.
3. *Das logische Bild der Tatsachen ist der Gedanke*. A logical picture of facts is a thought.
4. *Der Gedanke ist der sinnvolle Satz*. A thought is a proposition with a sense. ce.

TRACTAUS LOGICO-PHILOSOPHICUS

5. Der Satz ist eine Wahrheitsfunktion der Elementarsätze. A proposition is a truth-function of elementary propositions.

6. Die allgemeine Form der Wahrheitsfunktion ist: $[\bar{p}, \bar{\xi}, N(\bar{\xi})]$ *. Dies ist die allgemeine Form des Satzes*. The general form of a truth-function is: $[\bar{p}, \bar{\xi}, N(\bar{\xi})]$. This is the general form of a proposition.

7. Wovon man nicht sprechen kann, darüber muß man schweigen. What we cannot speak about we must pass over in silen

TRACTAUS LOGICO-PHILOSOPHICUS

1* Die Welt ist alles, was der Fall ist.

1.1 Die Welt ist die Gesamtheit der Tatsachen, nicht der Dinge.

1.11 Die Welt ist durch die Tatsachen bestimmt und dadurch, dass es a l l e Tatsachen sind.

1.12 Denn, die Gesamtheit der Tatsachen bestimmt, was der Fall ist und auch, was alles nicht der Fall ist.

1.13 Die Tatsachen im logischen Raum sind die Welt.

1.2 Die Welt zerfällt in Tatsachen.

1.21 Eines kann der Fall sein oder nicht der Fall sein und alles übrige gleich bleiben.

2 Was der Fall ist, die Tatsache, ist das Bestehen von Sachverhalten.

2.01 Der Sachverhalt ist eine Verbindung von Gegenständen. (Sachen, Dingen.)

2.011 Es ist dem Ding wesentlich, der Bestandteil eines Sachverhaltes sein zu können.

2.012 In der Logik ist nichts zufällig: Wenn das Ding im Sachverhalt vorkommen k a n n, so muss die Möglichkeit des Sachverhaltes im Ding bereits präjudiziert sein.

2.0121 Es erschiene gleichsam als Zufall, wenn dem Ding, das allein für sich bestehen könnte, nachträglich eine Sachlage passen würde.

Wenn die Dinge in Sachverhalten vorkommen können, so muss dies schon in ihnen liegen.

(Etwas Logisches kann nicht nur-möglich sein. Die Logik handelt von jeder Möglichkeit und alle Möglichkeiten sind ihre Tatsachen.)

* Die Decimalzahlen als Nummern der einzelnen Sätze deuten das logische Gewicht der Sätze an, den Nachdruck, der auf ihnen in meiner Darstellung liegt. Die Sätze n.1, n.2, n.3, etc., sind Bemerkungen zum Satze No. n; die Sätze n.m1, n.m2, etc. Bemerkungen zum Satze No. n.m; und so weiter.

TRACTAUS LOGICO-PHILOSOPHICUS

- Wittgenstein proclaimed that the only genuine propositions, namely, propositions we can use to make assertions about reality, were empirical propositions, that is, propositions that could be used correctly or incorrectly to depict fragments of the world. Such propositions would be true if they agreed with reality and false otherwise (4.022, 4.25, 4.062, 2.222). Thus, the truth value of an empirical proposition was a function of the world.

TRACTAUS LOGICO-PHILOSOPHICUS

Accordingly, mathematical propositions are not real propositions and mathematical truth is purely syntactical and non-referential in nature. Unlike genuine propositions, tautologies and contradictions (and Wittgenstein claimed that all mathematical proofs and all logical inferences, no matter how intricate, are merely tautologies) "have no 'subject-matter'" (6.124), and "say nothing about the world" (4.461).

TRACTAUS LOGICO-PHILOSOPHICUS

Mathematical propositions are “pseudo-propositions” (6.2) whose truth merely demonstrates the equivalence of two expressions (6.2323): mathematical pseudo-propositions are equations which indicate that two expressions are equivalent in meaning or that they are interchangeable. Thus, the truth value of a mathematical proposition is a function of the idiosyncratic symbols and the formal system that encompasses them.

The Middle Period

The middle period in Wittgenstein's philosophy of mathematics is characterized by *Philosophical Remarks* (1929-1930), *Philosophical Grammar* (1931-1933), and *Remarks on the Foundations of Mathematics* (1937-1944).

One of the most crucial and most pivotal aspects of this period is the (social constructivist) claim that "we make mathematics" by inventing purely formal mathematical calculi.

While doing mathematics, we are not *discovering* preexisting truths

" that were already there without one knowing."

The Middle Period

We use *stipulated* axioms (Wittgenstein, 1975 section 202) and syntactical rules of transformation to *invent* mathematical truth and mathematical falsity (Wittgenstein 1975 Section 122).

That mathematical propositions are pseudo-propositions and that the propositions of a mathematical calculus do not refer to anything is still prevalent in the middle period:

"Numbers are not represented by proxies; numbers are there ."

The Middle Period

Thus, this period is characterized by the principle that mathematics is a human invention. Mathematical objects do not exist independently. Mathematics is a product of human activity.

"One cannot discover any connection between parts of mathematics or logic that was already there without one knowing" (Wittgenstein 481)

The entirety of mathematics consists of the symbols, propositions, axioms and rules of inference and transformation.

Philosophical Investigations (*Philosophische Untersuchungen*)

The later Wittgenstein, namely the Wittgenstein of *Philosophical Investigations*, repudiated much of what was expressed in the *Tractatus Logico-Philosophicus*. In *Philosophical Investigations*, language was no longer a considered to be delineation but an implement. The meaning of a term cannot be determined from what it stands for; we should, rather, investigate how it is actually used.

Philosophical Investigations (*Philosophische Untersuchungen*)

Whereas the *Tractatus* had been an attempt to set out a logically perfect language, in *Philosophical Investigations* Wittgenstein emphasized the fact human language is more complex than the naïve representations that attempt to explain or simulate it by means of a formal system (Remark 23). Consequently, he argued, it would be erroneous to see language as being in any way analogous to formal logic.

Philosophical Investigations (*Philosophische Untersuchungen*)

Philosophical Investigations (*Philosophische Untersuchungen*)

Wittgenstein of *Philosophical Investigations* held that language was not enslaved to the world of objects. Human beings were the masters of language not the world. We chose the rules and we determined what it meant to follow the rules.

One might say that *Tractatus* is modernist in its formalism while the *Investigations* anticipates certain postmodernist themes

Philosophical Investigations (Philosophische Untersuchungen)

- There is disagreement on whether later Wittgenstein follows from middle Wittgenstein as is claimed by Wrigley (1993) or Rodych (2000) or if it is significantly different than that as claimed by Gerrard (1991) or Floyd (2005).
- As far as his philosophy of mathematics is concerned, we claim that there is at least one persistent thesis in all three periods. This, the most enduring constant in Wittgenstein's philosophy of mathematics is the claim that mathematics is a human invention. Just like the early and middle Wittgenstein, the late Wittgenstein also claims we "invent mathematics" (Wittgenstein 1956, I, 168; II, 38, V, 5, 9, 11) and thus

"… the mathematician is not a discoverer, he is an inventor" (Wittgenstein 1956, Appendix II, 2).

SOCIAL CONSTRUCTIVISM AND WITTGENSTEIN

The social constructivist thesis, as pronounced by Ernest, is that "mathematics is a social construction, a cultural product, fallible like any other branch of knowledge" and that "the justification of mathematical knowledge rests on its quasi-empirical basis."

The fact that the second claim, in addition, to Lakatos, et al., can also be attributed to Wittgenstein (1956), is the point of our paper.

SOCIAL CONSTRUCTIVISM AND WITTGENSTEIN

Indeed, the social constructivist epistemology, following Ernest (1999) “draws on Wittgenstein’s (1953) account of mathematical certainty as based on linguistic rules of use and ‘forms of life’, and Lakatos’ (1976) account of the social negotiation of mathematical concepts, results and theories.”

SOCIAL CONSTRUCTIVISM AND WITTGENSTEIN

For Wittgenstein, mathematics is a type of language game, for, as we shall show below, the formation of mathematical knowledge depends profoundly and organically upon dialogue that reflects the dialectical logic of academic discourse.

Wittgenstein's interest in the use of natural and formal languages in diverse forms of life (Wittgenstein 1953) prompts him to emphasize that mathematics plays diverse applied roles in many forms of human activity, such as sciences.

SOCIAL CONSTRUCTIVISM AND WITTGENSTEIN

As a natural consequence of its identification as a language game, mathematics ought to possess certain attributes, namely, rules, behavioral patterns, and linguistic usage that must be adhered to. As in any language, these syntactic rules would be crucial to sustain communication among participants.

The structure of these syntactic rules and their modes of acceptance evolve within linguistic and social practices. This, in turn, implies that consensus regarding the acceptance of mathematical proofs and consequently establishing theories arises from a shared language, a set of established guidelines, which are dependent upon social conditions.

SOCIAL CONSTRUCTIVISM AND WITTGENSTEIN

Interpreting mathematics as a language game also helps establish the social nature of mathematics. Language is crucial to social constructivism, as knowledge grows through language

“... the social institution of language . . . justifies and necessitates the admission of the social into philosophy at some point or other “(Ernest 1998, 131).

SOCIAL CONSTRUCTIVISM AND WITTGENSTEIN

Mathematics has many conventions which are, in essence, social agreements on definitions, assumptions and rules, that is to say, mathematical knowledge is a social phenomenon that includes language, negotiation, conversation and group acceptance. The conjectures, proofs and theories arise from a communal endeavor that includes both informal mathematics and the history of mathematics.

SOCIAL CONSTRUCTIVISM AND WITTGENSTEIN

As a result,

“Social constructivism accounts for both the 'objective' and 'subjective' knowledge in mathematics and describes the mechanisms underlying the genesis . . . of knowledge socially” (Ernest 1998, 136).

SOCIAL CONSTRUCTIVISM AND WITTGENSTEIN

Consequently, objective knowledge in mathematics is that which is accepted and affirmed by the mathematical community. This in turn implies that the actual objective of a proof is to convince the mathematical community to accept a claimed premise.

To this end, a proof is presented to a body of mathematicians. The proof is then carefully parsed, analyzed, and then accepted or rejected depending on the nonexistence or existence of perceptible flaws. If it is rejected, then a new and improved version is presented.

SOCIAL CONSTRUCTIVISM AND WITTGENSTEIN

The cycle continues in a similar fashion until there is agreement. Mathematical knowledge is, thus, tentative, and is incessantly analyzed. The process is indeed incessant because the guiding assumptions are based upon human agreements that are capable of changing.

This account of a social constructivist epistemology for mathematics overcomes the two problems that we identified with absolutism – infinite regress and consistency.

SOCIAL CONSTRUCTIVISM AND WITTGENSTEIN

Since the concepts of mathematics are derived by abstraction from direct experience of the physical world through negotiations within the mathematical community, mathematics is organically and inseparably coalesced with other sciences through language. Mathematical knowledge - propositions, theorems, concepts, forms of mathematical expressions - is constructed in the minds of individual mathematicians, participating in language games, in other words,

… mathematics is constructed by the mathematician and is not a preexisting realm that is discovered (Ernest 1998, 75).

SOCIAL CONSTRUCTIVISM AND WITTGENSTEIN

Consequently, the unreasonable effectiveness of mathematics is a direct result of the fact that it is built into language that is to say, to paraphrase Ernest, it derives from the empirical and linguistic origins and functions of mathematics.

The apparent certainty and objectivity of mathematical knowledge rests on the fact that mathematical symbolism is a sophisticated extension of written language - the rules of logic and consistency which pervade natural language form the crux of the objectivity of mathematics.

SOCIAL CONSTRUCTIVISM AND WITTGENSTEIN

In other words, as Wittgenstein noted, mathematical truths arise from the definitional truths of natural language, acquired by social interaction

The truths of mathematics are defined by implicit social agreement - shared patterns of behavior - on what constitute acceptable mathematical concepts, relationships between them, and methods of deriving new truths from old. Mathematical certainty rests on socially accepted rules of discourse embedded in our forms of life (Wittgenstein, 1956).