Stress On Analytical Reasoning (SOAR)

Suggested Background Modules for College Freshmen

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Presented at Spring 2012 meeting of MD/DC/VA Section of the Mathematical Association of America (MAA),

Mostly excerpts from http://www.math.umd.edu/~jnd/SOAR.html

Problem 1 on Speed. [Math SAT] "How many MINUTES are required for a car to go 10 miles at a constant speed of 60 miles per hour?"

(A) 600; (B) 100; (C) 60; (D) 10; (E) 6.

Only about half of students answered correctly**.**
[Item#5 of Section 7 of the May 2000 SAT Math test.]
Why is this Problem on Speed so "difficult"? Middle school students do know that {Distance} equals {speed} x {time} and how to convert from hours to minutes.
But doing *both* in a *single* problem largely falls through the cracks of the K-12 Math curriculum.

Problem 2. It is a fact that fat has 9 calories per gram and protein has 4 calories per gram. If a piece of meat consists of 100 grams of protein and 10 grams of fat, how many calories does it have altogether? (Answer: 490 calories; not a trick question)

Class time is used for instruction on how to do Problem 2 in "Nutrition 101" on my campus.

Problem 3. [NAEP Grade 8 2005]: There were 90 employees in a company last year. This year the number of employees increased by 10 percent. How many employees are in the company this year?

A) 9 B) 81 C) 91 D) 99 E) 100

Only half of Grade 8 *Algebra* students answered correctly [lucky guessers included].
[www.washingtonpost.com/wp-dyn/content/article/2008/09/21/AR2008092101813.html]

Percents are *not* in high school math curriculum. On my campus, students' eyes glaze over, when a professor mentions percents in sociology class.

The new national Common Core (CC) math curriculum includes about two weeks on percents, easily forgotten by the time students enter high school. I had suggested that students knowing 50% = ½ be included in CC Math. *Not* accepted, but being able to convert 50% to 50/100 was added.

These types of "Two operation" word problems fall thru the Math curriculum cracks; many students do not receive instruction on how to do them. A natural consequence is that the big bugaboo for students is word problems.

National Center for Education Statistics: " … far fewer [Americans] are leaving higher education with the skills needed to comprehend routine data, such as reading a table about the relationship between blood pressure and physical activity, … 'What's disturbing is that the assessment is [designed] ... to test your ability to read labels,'

Problem 4. 1 cubic yard = ? cubic feet
20% Correct on Calculus I Posttest (UT-Chattanooga)
[http://www.statlit.org/pdf/2009DarkenMAA.pdf]

Problem 5. (**Average Speed**) We flew from Denver to Boston at an average speed of 500 MPH; we returned from Boston to Denver at an average speed of 400 MPH. What was our average speed for this round-trip? WRONG answer. 450 MPH

- Plan A. Students learn all this in MS.
- Plan B. Students learn all this in HS.
- Plan C. Students learn all this as college freshmen.

Let the buck *stop* here in college freshmen year with very basic Stress On Analytical Reasoning (SOAR) courses. Have a subsection of the Math placement exam, devoted to word problems, with its own score. Also expand remedial courses to include "Two operation" word problems.

→ A SOAR basic math course's syllabus might include topics in
(*) non-trivial, multi-step Arithmetic and Algebraic word problems, with Stress On Analytical Reasoning (SOAR) as well as the Algebra word problems common to American high school Algebra books a half century ago.

(*) Arithmetic-level statistics, including knowledge and understanding of averages, medians, percentiles, box and whisker diagrams; also being able to read and draw graphs, charts and tables as well as proficiency with percents and decimals. Percentiles *not* included in CC Math.

→ A SOAR math course in logical reasoning and deductive logic might include
(*) basic theory of sets (Venn Diagrams and Boolean Algebra) .
(*) a deductive-proof-of-a-theorem each day (like the high school Euclidean geometry course of a half century ago)
(*) mathematical induction.

Prof. Barry Simon in "A Plea in Defense of Euclidian Geometry ", "mourned this loss of what was a core part of education for centuries." as he noted "what is really important is the exposure to clear and rigorous arguments. ... They can more readily see through the

faulty reasoning so often presented in the media and by politicians". [February 6, 1998 Los Angeles Times article.]
Jerry: Also, they would have less difficulty adjusting to and understanding college courses.

→ A SOAR Algebraic word problem course for potential STEM majors might include setting up the Algebraic formula for the maximum-minimum word-problems in the calculus textbook. (*No* calculus required. Could be added to Pre-Calculus course.)

→ A SOAR physics course's syllabus would include basic background in work, energy and power, speed and relative speed, density and pressure, and measurement; all topics which appear in newspapers. This would be an Arithmetic and Algebra I-based science course. This will provide "real world" problems as well as "real world" opportunities for estimation and use of measurement. *Better* yet, include these topics in middle school science. Warning: The National Research Council's (NRC) new Framework for K-12 Science Education omits the concept of work and the formula for kinetic energy [(1/2) mv^2], even in its section on energy.
["Arithmetic and Algebra Avoidance in [high school] Science and Statistics " by Jerome Dancis. This is my September 16, 2011 presentation to President [Obama's] Council of Advisors on Science and Technology (PCAST) [Reprinted in Education Views at http://educationviews.org/2011/10/26/arithmetic-and-algebra-avoidance-in-science-and-statistics/]

→ All these courses should provide instruction in reading comprehension and following directions.
Virginia Anderson (Towson State University, MD) provides extensive training to her college biology students in the reading and drawing of tables, graphs and charts Students should learn the difference between a statement and its converse and should not expect one to imply the other.

Writing and speaking mathematics. All these courses should include instruction on how to write and speak mathematics coherently, clearly, comprehensively, logically, accurately and precisely without being cryptic, vague, ambiguous, or obscure as well as how to distinguish a correct mathematical argument from an *incorrect or incomplete* mathematical argument. Even though writers of K-12 textbooks appear *not* to have such training.

Students need to be disabused of: "Death Valley is −282 feet *below* sea level." This is common in middle school math textbooks. ["**[Common] Missteps in Mathematics Books**" in the section "Fallacies, Flaws, and Flimflam" of The College Mathematics Journal (November 2008).]

Three of the next five "slides" are from Act II of Dr. David Kung's workshop (yesterday): "[Middle School] Mathematics for Social Justice". He asks college students to use Arithmetic to analyze remarks from newspapers and public debates.

Kung's workshop: "Sample Activity:
Annual Hospital Report
Administrator: 90% of patients who spend the night check out within a week.
Nurses: 80% of the patients who stayed last night have been here over a year!
Q: Can they both be right?

Both can be right!

[Picture ten beds in a hospital. There are 8 "permanent" patients; the remaining two beds were empty on some nights; the other nights they were used by 72 short term patients who stayed 6 days (or less) each.]"

"Similar [seemingly contradictory unstated "long-term" versus "short-term"] statistics hold for people:

- welfare
- unemployed
- without health insurance

Your agenda determines the statistics you use!"

Jerome Dancis: This is similar to the students' versus the colleges' view of average class size.
The college's ad says that the average class size is about 31 students.
The students complain that they see (on average) more than a hundred fellow students in their classes.

How can both be correct?

Consider a school with 650 math students divided among 21 math classes; 20 have 20 students each and one has 250 students.
Then the average class size is 650/21 or about 31 students. This is the number that appears in the school's publications.
But, from the students' prospective, there are 400 students in classes of 20 and 250 students in a class of 250.
The average number of classmates for each of these 650 students, is more than a hundred.

Back to Kung's workshop: "What our students **need**:
McCain: "Sen. Obama's secret that you don't know is that his tax increases will increase taxes on **50%** of small business revenue."
Obama: "Only a few percent of small businesses make more than $250,000 a year. So the vast majority of small businesses would get a tax cut under my plan." "**98% percent** of small businesses make less than $250,000"

Can you reconcile these statements or is one of them lying?"

Dancis: All college graduates (and especially newspaper reporters) should be able to do the simple, albeit mildly tricky Arithmetic thinking needed for analyzing many public

statements by politicians and others. This includes easily reconciling many simple supposedly contradictory statements in public debates, as well as seeing through the spin of self-serving statements. All (not just a few) students should learn this; this learning might occur in a college political science class, better yet in a high school social studies class. Until this happens, it will be useful to include examples in elementary Math classes. This is very important.

Students might be trained to do many of the problems in my reports, "Reading Instruction for Arithmetic Word Problems: If Johnny can't read well and follow directions, then he can't do math" and "Algebraic Word Problems" on my Math Education Website: www.math.umd.edu\~jnd

Average Speed Problem 5. We flew from Denver to Boston at an average speed of 500 MPH; we returned from Boston to Denver at an average speed of 400 MPH. What was our average speed for this round-trip?

- WRONG answer. 450 MPH