Mathematical Association of America
MD-DC-VA Section, April 15-16, 2011
Randolph-Macon College
Abstracts

Invited Addresses

FRIDAY WORKSHOP

Richard Gillman, Valparaiso University

A Game Theory Approach to Quantitative Literacy

4:00 pm, Copley 101

This workshop explores the ways in which game theory topics can be used to motivate a general audience of students to review basic mathematics skills, and to utilize them to solve real problems from a quantitative perspective. Over the course of the two hours, participants play deterministic and coalition games while exploring key solution concepts.

BANQUET ADDRESS

Sommer Gentry, United States Naval Academy

Faster, Safer, Healthier: Adventures in Operations Research

8:00 pm, Alumni Gym

While mathematical advances of all sorts have impacted our world for the better, operations research is a branch of mathematics that is expressly focused on applying advanced analytical methods to help make better decisions. Operations researchers have eased traffic jams by closing selected streets, and gotten packages to you more quickly by planning U.P.S. routes with fewer left turns. Operations researchers have shown which personal decisions are the leading causes of death, and planned maintenance schedules to minimize bridge collapses. The mathematical tools of operations research, like using random numbers to simulate a range of outcomes when some data are unknown, or finding clever algorithms that shortcut the need to try every possible decision in order to find the best one, can be recycled to solve problems everywhere in our world. In this talk, I will describe some of my O.R. forays into far-flung fields, and tell my favorite stories about O.R.

SATURDAY INVITED ADDRESSES

Sarah Greenwald, Appalachian State University

Rubik's Cube Games on Spheres: Geometry of Spherical Orbifolds

9:20 am, Blackwell Auditorium

We'll slice up basketballs in order to form new spaces like footballs and triangular pillows, and then look at the geometry of the resulting spaces, called orbifolds. Orbifolds furnish a natural starting point for the study of singular spaces and they are especially of interest to mathematicians and physicists. Diverse applications of orbifolds include connections to crystallography, string theory and music theory. Many results, such as those requiring local analysis, generalize easily to the orbifold setting, but most global results do not. Imagine a spherical Rubik's game where you can rotate spherical triangles on the surface of the sphere. This game exists and is called the Impossiball and we'll use it to help understand orbifolds, as we look at lots of examples and results related to the diameter, Euler characteristic, and spectrum.

Richard Gillman, Valparaiso University

Everyday Questions, Not-So-Everyday Mathematics

3:35 pm, Copley 100

The world is full of un-explored mathematical problems. This talk presents the stories of three problems that the presenter found in his everyday world and investigated with undergraduate research partners. One is solved completely, one quickly reaches deep and un-explored mathematical territory, and the third, while not solved, opens many paths for further exploration.

Contributed Papers by Author

Ezra Brown, Virginia Tech
Adrian Rice, Randolph-Macon College
Elliptic Curves, Weierstrass, Doughnuts, and 49/20: Why Ellipses Are Not Elliptic Curves (Part III)
9:10 am, Copley 101
Why is an elliptic curve like a doughnut, why is Weierstrass involved, and what is so special about the number 49/20? In Part III of "Why Ellipses Are Not Elliptic Curves," we supply the answers to these questions. This talk is independent of Parts I and II.

Hongwei Chen, Christopher Newport University
When Harmonic meets Fibonacci
8:20 am, Fox 106
The Riemann series theorem says that the terms of a conditionally convergent series may be rearranged in such a manner that the resulting series converges to any prescribed value. What about grouping the harmonic series into Fibonacci length blocks? Alternating the harmonic series in Fibonacci length blocks? Rearranging and alternating the harmonic series into separate blocks of even and odd terms of Fibonacci length? See what this talk discovered.

Jerome Dancis, University of Maryland
Statistics versus Endless Algebra for Grade 12 Math
3:05 pm, Copley 132
Debate: Statistics or Algebra or what for Grade 12 Math? That is the question we will debate starting with some comments from the NCTM president's article, "Endless Algebra—the Deadly Pathway from High School Mathematics to College Mathematics", the MAA Past president David Bressoud's half-rebuttal: "The End of Algebra?" and my reports "College Readiness -- A Simple Description" and "College Success –- A College Professor's Perspective".

Zhou Dong, Montgomery College
Three chords and a circle
3:05 pm, Copley 133
In this talk we present a probability problem related to the geometry of chords on a circle. Given a circle, a (unique) inscribed equilateral triangle, and a randomly placed chord, what is the probability that the chord is longer than the side of the equilateral triangle? We will explore how different readers can produce different but equally plausible solutions to this problem.

Andy Dorsett, Wolfram Research
Mathematica 8 for Education
8:45 am, Fox 107
This talk illustrates capabilities in Mathematica 8 that are directly applicable for use in teaching and research on campus. Topics of this technical talk include: Free-form linguistic input, 2D and 3D visualization, Dynamic interactivity & On-demand scientific data, Example-driven course materials, Symbolic interface construction. Current users will benefit from seeing the many improvements and new features of Mathematica 8, but prior knowledge of Mathematica is not required.

Bryan Faulkner, Ferrum College
Introductory Algebra Applets with GeoGebra
3:05 pm, Fox 107
GeoGebra is a freely available mathematics software. GeoGebra applets can be created which run on essentially any computer using a web browser. Therefore, once an applet is created it can be sent electronically to students. The goal of this project is to write a small library of applets and assignments. The intention is that the applets will be an aid for the assignments. This presentation will include a brief introduction to GeoGebra. Several GeoGebra applets will be discussed.

Rebecca Field, James Madison University
Elizabeth Arnold, Stephen Lucas, and Laura Taalman, James Madison University
Which Sudoku boards are essentially different?
2:15 pm, Fox 108
There are two basic ways to rearrange a Sudoku board to produce another Sudoku board which is essentially the same as the initial board. The first way is by using physical symmetries such as rotating a board 180 degrees or switching the bottom two rows. The second is by relabeling the entries, such as turning all of the ones into twos and vice versa. We will discuss the ways that these two types of Sudoku symmetries interact for the special case of Shidoku, the four by four equivalent of Sudoku. We will also discuss how these are used, along with Burnside's lemma, to count the number of essentially different Sudoku/Shidoku boards.

Raymond Fletcher, Virginia State University
Perfect Polygons with Irreducible Cubic Envelope
8:20 am, Copley 101
Let P be a set of n points in the plane, labeled with Z(mod n) and let W(k) denote the set of lines {xy : x+y = k (mod n)}. If for each k in Z(mod n) the lines in W(k) are concurrent, then we call P a perfect n-gon. The point X(k) of concurrence of the lines in W(k) is called a perspective point of P. The combined set of vertices and perspective points of P lie on a cubic curve C, the cubic envelope of P. Here we focus on the case in which C is irreducible and an algebra (C,*) can be defined on the points of C by setting x*y = the third point on C, (besides x,y) which lies on xy. If P has all its vertices on the same branch of its cubic envelope then we call P contiguous. We show, for example, that every perfect contiguous n-gon with irreducible cubic envelope is a coset of a certain subalgebra of (C,*).

Brian Heinold, Mount St. Mary's University
Iterating a discontinuous function
3:05 pm, Copley 101
In this talk we will look at iterating a certain discontinuous function that can be thought of either in terms of rotations and dilations or in terms of a simple formula in the complex plane. From this iteration we create images by coloring points according to how many iterations it takes until the iterates appear to converge. The resulting images are symmetric, beautiful, and occasionally peculiar, being different from most fractals you may be familiar with.

Heidi Hulsizer, Hampden-Sydney College
Resolutions of Determinantal Ideals
9:10 am, Fox 106
Determinantal ideals have been studied for over a century and they continue to be of interest to mathematicians. The structures of several resolutions (complexes) of determinantal ideals have been determined. The hope was that the structure of all resolutions of this form could be found and that they would not depend on the characteristic. This possibility was crushed when, in 1990, Mitsuyasu Hashimoto showed that this was impossible. We will discuss the structure for the longest possible resolution that does not depend on the characteristic of the field. To describe the resolution we use what is called tableau notation. This notation is used in other branches of mathematics and it works well to define the basic elements involved in the maps of the resolution.

Anna Johnston, Embedics, LCC
Truncated Taylor Series, Interpolation, and the Chinese Remainder Theorem
8:20 am, Fox 107
The Chinese remainder theorem is a ring isomorphism theorem best known over the integers. Over polynomial rings the versatility and multitude of applications are even greater. This paper looks at one of these applications: using the Chinese remainder theorem to derive Taylor series approximations. With Taylor series rooted firmly in continuous mathematics and the Chinese remainder theorem in discrete, this derivation bridges the two sides and sheds a different light on what Taylor series approximations represent.

Abstracts

Dan Kalman, American University
Newton's Identities via Reverse Long Division
2:15pm, Fox 107
Although it is not generally an easy matter to find the roots of a polynomial, certain functions of the roots ARE easy to find, including for example the sum of the roots, the product of the roots, and the sum of the reciprocals of the roots. Newton's name is attached to a series of identities that specify sums of integer powers of the roots (although he wasn't the first to discover such identities). In this talk I will show how to generate these sums of powers using a simple long division process.

Robb Koether, Hampden-Sydney College
How to play a game when you do not know what game you are playing
2:15 pm, Copley 101
Suppose two players (row and column) are playing one of two 2x2 games G and G'. If they both know which game they are playing, then there is no problem. So we will first suppose that the column player knows which game they are playing, but the row player knows only probability that they are playing game G. Then we will suppose that neither player knows which game they are playing, but they know only the probability that it is game G. In both cases we will present a method to find optimal strategies for both players.

Brian Lins, Hampden-Sydney College
Liberal Arts Mathematics on a Logarithmic Scale
3:05 pm, Fox 108
I will demonstrate the use of logarithmic scales to illustrate concepts in a math for liberal arts course. Applications such as slide rules, Benford's law, and data presentation will be mentioned. We will also discuss connections with compound interest and the Fibonnaci sequence.

John Lorch, Ball State University
Ellen Weld, Ball State University
Modular Magic Sudoku
2:40 pm, Fox 108
A modular magic Sudoku square is a Sudoku square with symbols in {0,1,...,8} such that rows, columns, and diagonals of each subsquare add to zero modulo nine. We count these Sudoku squares by using the action of a suitable symmetry group and we also describe maximal orthogonal families. The collection of modular magic Sudoku squares seems well suited for testing counting techniques designed for Sudoku squares possessing rows/columns/diagonal sum conditions on their subsquares.

Dante Manna, Virginia Wesleyan College
Expected Independence Polynomials
9:10 am, Copley 200
The independence polynomial of a (finite) graph is the generating function for the number of independent sets of each cardinality. Assuming that each possible edge of a complete graph of order n is independently operational with probability p, we consider the expected independence polynomial. We show here that for all fixed p in (0,1) the expected independence polynomials of complete graphs have all real, simple roots. Moreover, we find upper and lower bounds for the expected number of independent sets in a graph of order n, and thereby provide bounds and an asymptotic for the average number of independent sets in a graph of order n.

Roland Minton, Roanoke College
Some Data Mining Cave-Ins
2:40 pm, Copley 101
The PGA Tour's ShotLink system produces a massive data set detailing nearly every shot taken in PGA Tour events, with the location of the ball measured to the inch. ShotLink provides innumerable opportunities for data analysis, with as many opportunities for mistakes and misinterpretations. A small sample of odd preliminary findings and their resolutions will be presented.

John Nolan, American University
Math for America DC
2:40 pm, Fox 107
We give an overview of the Math for America Program, a nationwide effort to train and place talented math students as secondary math teachers in high need schools. We will focus on the Washington, DC program, which is now completing its second group of students.

Marcus Pendergrass, Hampden-Sydney College
M. Leigh Lunsford, Longwood University
Galton Meets Bayes: Prior Beliefs Inform The Scientific Method
2:40 pm, Fox 106
While statistics is central to the "scientific method," traditional parameter estimation and hypothesis testing techniques limit the researcher's ability to factor prior beliefs into the conclusions they draw from experiments. In this talk we will argue that Bayesian inferential techniques, which are based on Bayes' Theorem from probability, more accurately reflect the scientific method. In addition, we will demonstrate the fundamental concepts of Bayesian inference through real-time data collection and analysis of an actual (not computer-simulated) Galton Board. Through this demonstration, attendees will see how Bayesian inference provides a beautiful and unifying framework for parameter estimation, maximum likelihood estimation, and hypothesis testing.

Adrian Rice, Randolph-Macon College
Ezra Brown, Virginia Tech
A Tale of Two Surfaces: Why Ellipses Are Not Elliptic Curves (Part II)
8:45 am, Copley 101
Elliptic curves are a fascinating area of algebraic geometry with important connections to number theory, topology, and complex analysis. As their current ubiquity in mathematics suggests, elliptic curves have a long and fascinating history stretching back many centuries. This paper presents a survey of key points in their development, via elliptic integrals and functions, closing with an explanation of why no elliptically-shaped planar curved line may ever be called an elliptic curve. This talk is independent of Parts I and III.

Robert Sachs, George Mason University
Discussion on Survey – Transition from k-12 to College
2:40 pm, Copley 132
The session will ask for input concerning a soon-to-be-distributed online survey. The questions will cover a range of issues all concerning the transition from HS to college mathematics. This involves multiple subpopulations from remediation to advanced placement.

Michael Smith, Hollins University
Cone Tipping: A Cooperative Initiative in Abstract Algebra
2:15pm, Copley 132
This talk presents a variation of a common team building initiative known as "Key Punch." By completing this game, students collect numerical data which leads them to conjecture patterns about orders of elements and subgroup structure of Z_{20} in particular, and Z_{n} (and therefore all finite cyclic groups) in general.

James Sochacki, James Madison University
The Newton Cannon Ball Problem is a Polynomial Problem
2:15 pm, Fox 107
I will show that the problem of determining the motion of cannon ball shot from the North Pole is a two body problem that can be posed as a polynomial problem. We then show if the moon is added to this problem results are substantially different, but that the problem is still polynomial. I will then demonstrate that most dynamical problems involving a force field are polynomial problems.

Brian Sutton, Randolph-Macon College
Conformal Maps for Numerical Analysis
3:05 pm, Fox 106
This talk will advocate Chebyshev-based methods for numerical analysis in the classroom. First, we consider a neighborhood of the unit circle in the complex plane. Then, we conformally map an interval of the real line onto the unit circle, obtaining real numerical analysis as a special case. The resulting methods are stable and efficient and provide gateways to complex analysis, Fourier analysis, and functional analysis. Many of the results are old ones--S. N. Bernstein proved an important convergence result in 1912--but the approach is experiencing a resurgence through the Chebfun project at Oxford University.

Bruce Torrence, Randolph-Macon College
The Viewable Sphere: Mathematics Meets Photography
9:10 am, Fox 107
In a certain sense, the "ideal" camera would capture a full panoramic scene from a single vantage point. That is, it would produce a spherical image where the vantage point is at the center of the sphere, and the color of each point on the sphere is determined by the color of the object first encountered by the ray of light traveling from the center of the sphere through that point. Of course, there are good reasons why we don't routinely see such photographs. Cameras can only capture a small portion of the "viewable sphere," and printers, well they just don't print on spheres. The first problem is easily solved with digital imagery; it is a relatively simple matter to "stitch" several images together to create a full 360 x 180 degree panorama. The second problem can be addressed with mathematics. Just as cartographers developed methods to display a spherical planet on a flat surface, similar methods can be used to render the viewable sphere. We will see several well-known projections in a new light: equirectangular, stereographic, Mercator, Lagrange. Moreover, we will see how conformal maps between regions in the complex plane relate to this endeavor.

Kenneth Whipple, Georgia State University (retired)
An Efficient Way to Generate Permutations
8:45 am, Fox 106
The problem of generating all permutations of order n in such a way that each permutation is a simple swap of two elements of the preceding permutation is considered. The solution involves keeping a count of the permutations and using only the count to determine the necessary swap. The relationship between a progressive based number system and the permutations is presented.

Student Abstracts by Author

Patricia Bellew (Senior), James Madison University
Dynamical Periodic Orbits
8:20 am, Copley 133
We analyzed the properties of quadratic planar ordinary differential equations with periodic orbits. Also allowing for imaginary numbers, we found the equilibria, the stability, and looked at solutions to check for periodicity. After keeping a journal of all results, we then applied these concepts to do an in-depth study of the Solow economic growth model. Adjusting the original model to analyze subjects of interest, such as technological and educational changes within an economy, we used a numerical ODE solver to do a quantitative analysis of these modified models.

Abstracts

Kathryn Christian (Senior), University of Mary Washington
Mathematical and Numerical Solutions for a Heat Conduction Model
2:15 pm, Copley 133
We use differential equations to model the conduction of heat energy in 1D and 2D regions. For the steady-state heat conduction problems with Dirichlet boundary conditions, we find the exact solution to the 1D problem in an interval, and we find the exact solution to the 2D problem inside a rectangle using the method of separation of variables. There are two different forms of the solution to the problem in 2D, one using four subproblems and one using two. Computationally, we find a method to approximate the solution to the 1D problem using polynomial basis functions. We run numerical experiments to test the accuracy of the numerical solution for the 1D problem and to compare the computational work necessary to calculate the four-subproblem and two-subproblem solutions at different points inside rectangles.

Kevin Doubleday (Senior), University of Mary Washington
Application of Markov Chains to Stock Trends
9:10 am, Copley 133
The Dow Jones Industrial Average is analyzed with a discrete time stochastic model, namely a Markov Chain. Two models are highlighted, where the DJIA is considered as being in a state of 1) gain or loss and 2) small, moderate, or large gain or loss. A portfolio of five stocks is then considered and two models of the portfolio much the same as those for the DJIA. These models are used to obtain transitional probabilities and steady state probabilities, which are in turn tested. Our results show that the steady state distributions for all four models closely resemble those determined by examining data not included in the model. The first model of the DJIA correctly predicts a higher number of market gains than losses and the second model correctly predicts an approximate normal distribution for the steady state probabilities. We also found that the second model of the portfolio lacks a steady state probability vector.

Geoffrey Driskell (Senior), University of Mary Washington
A Comparison of Two Derivations of the Black-Scholes Option Pricing Model
8:45 am, Copley 133
One of the most widely used models in financial mathematics is the Black-Scholes Option Pricing Model. It has been the benchmark against which all other option pricing models have been judged since its introduction nearly forty years ago. The model can be derived in a number of different ways using mathematical techniques of varying sophistication. To gain a better understanding of this model we will explain and compare two of the most popular and widely used derivations of the model that use two completely different mathematical approaches, the Dynamic Hedging Strategy and Risk-Neutral Valuation. Throughout this process, concepts such as the Wiener Process, Ito's Lemma, Geometric Brownian Motion and, put-call parity that make up the foundation of Financial Mathematics will be introduced and applied to the practice of theoretical option pricing.

Caleb Gibson (Senior), Virginia Military Institute
Superbolas: Generalizing the Generalized Parabola
8:20 am, Fox 108
A parabola can be defined as the locus of all points equidistant from a point called the focus and a line called the directrix. In a recent paper we explored the consequences of allowing the directrix to be a more general curve. The resulting "generalized parabolas" have many interesting properties including a reflective property. In this talk we consider the results of also allowing the focus to be a curve. The resulting "superbolas" are, in general, much more complex than generalized parabolas, but they still retain the reflective property.

Natalie Horvath (Senior), Roanoke College
Should I Stay or Should I Go: An Optimal Solution to the Ride and Tie Problem
8:45 am, Fox 108
In this presentation we will discuss an optimal solution to the Ride and Tie problem in the context of both a mathematical and a statistical formulation. Ride and Tie is a distance race that involves two people and one horse. The race begins with one team member on foot and the other on the horse. At some point, before the finish line, the first team member ties the horse to a tree and runs to the finish line. The second team member runs until she/he arrives at the horse and then rides to the finish. The objective of the race is to get both participants to the finish line as quickly as possible. A natural question that arises is where is the optimal location to tie the horse? We present a mathematical formulation of the question when the rider has the option to tie the horse anywhere along the course. Next we move on to the more realistic case when there are only a limited number of trees to which the horse can be tied. In this formulation we describe an algorithm for deciding if the first rider should tie the horse to a given tree or continue riding.

Ryan Shifler (Senior), Salisbury University
Universal Groebner Bases of Circulant Polynomial Systems
9:10 am, Fox 108
In the study of linear algebra, a circulant system of equations occurs when the first equation generates each of the following equations by applying the same shift to the coefficients from one equation to the next. These systems have properties that linear systems do not have in general. They can be generalized to non-linear polynomial equations where circulant activity may exist with the coefficients and have the same kind of shifts as the linear case. Finding solutions to the non-linear polynomials is equivalent to finding the solutions of the ideal generated by polynomials. The solutions to the generating set can be obtained with a Groebner basis which can change under different monomial orderings in a polynomial ring. A Groebner basis is a computationally preferable generating set for the ideal in a polynomial ring, much like row reduced echelon form in the linear case. Buchberger's algorithm is implemented with the aid of a computer algebra system in order to see how different monomial orders affect the Groebner Basis. In the multivariable case, the number of monomial orders is infinite, although the number of reduced Groebner bases for a polynomial ideal is always finite. Investigating sufficient conditions for these non-linear systems to produce a nice Universal Groebner bases is the main focus of this research.

Kelsie Snyder (Senior), University of Mary Washington
c-Dominating Sets for Families of Graphs
8:45 am, Copley 200
The topic of domination in graphs has a rich history, beginning with chess enthusiasts in the 1850s determining how many queens are necessary to dominate an entire chessboard and continuing to current problems involving computer communication networks, social network theory, and other similar problems. We define a dominating set of a graph G to be a set of vertices of G such that every vertex of G is either in the set or adjacent to a vertex in the set. The domination number for a graph G is the size of a minimum dominating set. Determining the domination number of graphs can prove highly useful in solving many types of problems, and recent studies of dominating sets reflect this. We focus on describing various families of graphs in terms of bounds on the domination number. Although the computation of dominating sets for arbitrary graphs is an NP-complete problem, it is possible to compute certain bounds on the domination number for certain families of graphs. We examine families of graphs, specifically the family of grids, and determine the bounds on domination number for these families.

Abstracts

Andrew Snyder-Beattie (Senior), University of Mary Washington
Dissecting Two Approaches to Energy Prices
2:40 pm, Copley 133
Geometric Brownian Motion (GBM) has been used to model the behavior of the world's energy markets, but some mathematicians posit that markets for oil are not well-modeled by stochastic processes with stationary independent increments. We confirm that GBM may be a problematic assumption for modeling the price of oil and that a standard Ornstein-Uhlenbeck process does a better job, implying that oil prices are mean-reverting. After isolating OPEC quota periods, we argue this behavior does not occur as a result of OPEC price controls.

James Street (Senior), Randolph-Macon College
Finding Complete Minor-Minimal Sets Using Δ –Y Moves
8:45 am, Copley 132
A graph H is a minor of a graph G if H is obtained from G by edge contractions and/or edge deletions. Many useful properties of graphs can be characterized by their complete minor-minimal sets. We'll explain what these are, and explore how so-called Δ–Y moves interact with certain graph properties and their minor minimal sets.

Guan Wang (Senior), Randolph College
Non-repetitive Sequences and the Tower of Hanoi
8:20 am, Copley 132
A non-repetitive sequence is a sequence that contains no identical adjacent segments. This concept was first introduced in the 1960s, and has ever since been explored. In 1996, it was proven that the sequence of optimal moves in the puzzle Tower of Hanoi (TOH) is always non-repetitive. This intriguing discovery has the potential to offer Mathematicians new angles to study non-repetitive sequence. Therefore, the objective of our research was to find out the essential elements of TOH that yield non-repetitiveness.

Jessica Zlotkowski (Senior), Longwood University
Area Ratios in Euclidean Geometry
9:10 am, Copley 132
Using interactive geometry software, we begin with a square, construct a specific quadrilateral in the interior of the square, and investigate the ratio of the area of the newly constructed quadrilateral to the area of the starting square. We generalize the starting figure from a square to a general convex quadrilateral and investigate the ratio of the areas of the resulting quadrilaterals.