MD-DC-VA Section MAA Spring 2009 Meeting at University of Mary Washington

Contributed Paper (CP) Faculty Abstracts

Developing a new mathematics travel course - challenges, rewards and lessons learned Chiru Bhattacharya, Randolph-Macon College CP6	Travel courses are becoming increasingly popular with students at liberal arts colleges. Randolph-Macon College recently offered a new mathematics travel course to India. I will share some of the experiences in developing the course, organizing the trip and teaching the math.
Tom Kirkman and his Electric Schoolgirls, Part II Ezra Brown, Virginia Tech CP3	The Kirkman schoolgirls problem asks for 7 distinct arrangements of 15 girls into 5 rows of 3 girls each with the condition that each girl walks in a row with every other girl exactly once. We previously talked about connections between this famous problem and algebraic number fields, and finite projective 3-space. This talk, in which we explore the connections with error-correcting codes and hat puzzles, is independent of the previous talk.
Applications of the Central Limit Theorem Owen Byer, Eastern Mennonite University CP2	This elementary talk will examine applications of the Central Theorem related to probability games (one gambling game and the board game Risk). Students should find them interesting and faculty should find them useful to use as class examples/homework problems.
Introducing Commutative Algebra to Academically Diverse Populations Boyd Coan, Norfolk State University CP4	This is a talk about course design. While learning about the subject of Commutative Algebra a natural question might arise. How should Secondary School majors be introduced to the topic? The target audience consists of undergraduates at colleges with an academically diverse population. Underestimating the difficulty of transporting these ideas to such a group can result in the abuse of the mathematical results. A typical topic in Pre-Calculus is Long Division of One Variable Polynomials over the Reals. Usually, it is taught by comparing it to the long division learned in grade school and emphasizing its algorithmic character. Later, in an Abstract Algebra course things take on a more theoretical nature requiring more deductive reasoning. To teach the general Division Algorithm for polynomials in several variables, however, a return to recursion and inductive reasoning is needed. After this, Grobner Bases and Buchberger's Algorithm might be introduced which allows the pursuit to learn Computational Commutative Algebra to continue.
On the summation of subseries in closed form Hongwei Chen, Christopher Newport University CP3	In this talk, we will reprove a series multisection formula, which enables us to sum subseries via the appropriate linear combination of the original series. In an effort to popularize and promote the formula, we will illustrate it by a variety of examples.
Thurston's Conjecture-Topology Change and the Universe George DeRise, Thomas Nelson Community College CP2	This will be a very intuitive talk connecting some of the ideas of Thurston's Geometrization Conjecture, topology change, and how these ideas can possibly model the expansion of the Universe.

Barycentric Coordinates and its Application Ming Fang, Norfolk State University CP6	In Euclidean coordinate system, we use two coordinates to describe a point in the plane. In this talk, we will discuss barycentric coordinates, which describe points in two dimensional triangle geometry by three coordinates associated with each vertex of the triangle. We will present the application of barycentric coordinates to numerical analysis.
A 200-Question, Campus-Wide Math Contest Brian Heinold, Mount St. Mary's University CP5	Last year at Mount St. Mary's, we held a campus-wide, 200-question math contest to celebrate the University's Bicentennial. Each week throughout the school-year we posted seven new problems, ranging from the easy, "What math word can be split into two words that might describe a man after a long day at the beach?" to the more mathematical, "How many integers between 1 and 10,000 contain the number 3?" Problems were designed to be fun and solvable, at least in principle, by a non-math major. People from all parts of the university participated - students, faculty, staff, administrators, and seminarians. This talk will give an overview of the contest and suggestions on how to run something similar at your school.
Sums of Reciprocals of Polynomials over the Field $F_2[x]$ Kenneth Hicks, Ohio University CP1	The well known Euler product formula relates the sum of reciprocals of integers to a product over inverse prime powers. In this talk it is shown that, over the binary field, the sum of the reciprocals of polynomials in $F_2[x]$ of degree n is equal to an inverse product over all irreducible polynomials of degrees 1 to n.
Fast Solvers for Models of Fluid Flow P. Aaron Lott, National Institute of Standards and Technology CP1	Numerical simulation provides insight into the effect physical parameters have on fluid flows under conditions that make physical experiments and theory intractable. However, these simulations are computationally demanding and in order to extend their applicability, highly scalable and efficient numerical methods are being developed. We discuss a novel block preconditioner based on domain decomposition and fast diagonalization that can be used to accelerate iterative solution methods. We then demonstrate how this technique provides an efficient means of simulating steady fluid flows.
Taylor Series Without High Order Differentiation Stephen Lucas, James Madison University CP1	Finding Taylor series of functions to high order can lead to very large and complicated derivatives. Here, we show how rewriting functions as solutions of differential equations leads to quick and easy series. A new algorithm for finding Bernoulli numbers will be presented, as well as the relationship to the Parker-Sochacki method for solving odes.
Novel Pedagogical Resources Based on Protein Structure Analysis Majid Masso, George Mason University CP3	Life science applications in undergraduate mathematics classes typically focus on ODE and PDE modeling of biological systems. Biomathematics courses may also illustrate the utility of graph theoretic, numerical, and statistical approaches in the study of DNA and RNA molecules. However, in-class examples and homework exercises based on protein structure analysis are rarely exploited despite an inherently large number of quantitative applications. This talk will begin with a basic

	overview of proteins and their 3D structures, followed by a description of one particular approach to protein structure analysis that draws on methods from computational geometry, finite mathematics, probability theory, computer programming, and statistical mechanics. The example provides a rich new source of pedagogical tools that are easily implemented in the mathematics classroom.
Untangle - Knots and Games Alex Meadows, St. Mary's College of Maryland CP6	We introduce a two-player game on presentations of the unknot, in which moves are Reidemeister moves. The goal is to classify the starting positions by the player (the first or second to move) who is able to win regardless of the other player's strategy. This is research that will be continued at an REU this summer.
Rating the PGA Golfers Roland Minton, Roanoke College CP6	Based on shot-by-shot data from the 2007 season, ratings of golfers in separate categories of putting, iron shots, drives and so on are computed, leading to an overall rating of the golfers. Guess who is number one?
Analyzing Self-Congruence Using Bristle Sets Steve Morse, retired CP4	Using the five regular Euclidean solids as exemplars, the paper applies Euler's Theorem on rigid rotations of the sphere to self-congruences of polyhedra. The set of axes of rotation are considered as a geometric object, the bristle set. The three bristle sets for the tetrahedron, octahedron, and icosahedron are displayed and analyzed.
Topological Consequences of an Algebraic Representation for P G. Edgar Parker, James Madison University CP3	P is the set of real-analytic functions each of which has the property that it is a component of the solution to a differential equation with a polynomial generator and initial conditions set at 0. The techniques of polynomial projection guarantee wide applicability of the approximation schemes that go with belonging to P. P has been shown to be algebraically closed under $+$, $*$, and $\circ$, and a proper sub-algebra of those operations on the set A, consisting of the real-analytic functions having 0 in their respective domains. In studying the algebras on A, classic power series operations remove the necessity to deal with domain considerations. In this talk, we review a representation of P in which, when studying the algebras on P, domain considerations can also be ignored. We then explore topological consequences for P that can be proven through the representation.
Fourier Transforms and Signal Processing : A Theme for an Applied Mathematics Course Marcus Pendergrass, Hampden-Sydney College CP4	Signal processing is replete with beautiful mathematical applications. It also offers a rich and intuitive context for teaching the Fourier transform. In this talk I will describe a course in Applied Mathematics that weaves together these two ideas. I will show how the applied context addresses some of the pedagogical challenges of teaching the Fourier transform, and I will describe a class project that used the Fourier transform to solve a nontrivial signal processing problem.

Jump Shot Mathematics Howard Penn, U.S. Naval Academy CP5	Cobie Bryant takes a 15 foot jump shot, releasing the ball from a height of 10 feet, and an angle of 60 degrees. What initial velocity is necessary to hit the shot? If he later takes the same shot at an angle of elevation of 30 degrees, what in the initial velocity? DO THESE SHOTS GO IN?
Computing Probability Generating Functions of Coin Flipping and Its Generalization Cherng-tiao Perng, Norfolk State University CP1	A classical probability question asks for the expected waiting time for flipping a coin (fair or not) until a series of consecutive k heads occur. Now instead of k heads, we can ask for the expected waiting time for a prescribed string such as HTHHTT (H means head and T means tail), and furthermore, the following more general setting: replacing coin flipping by taking a letter, one at a time, what is the expected waiting time until a prescribed string (a series of letters) is reached? Here we allow different probabilities for the occurrence of different letters. Our approach to this question is computational: we show that there exists a universal program taking as inputs the choice of letters with given probabilities and the prescribed string, and as output, returning the probability generating function for the waiting time. In particular, this solves the problem of finding the expectation and variance for the waiting time random variable.
On the Intersection of Domination and Experimental Mathematics Robert Rubalcaba, Department of Defense CP2	Inspired by the recent book ``The Computer as Crucible" by Borwein and Devlin, this talk is an example of experimental mathematics. Through the use computer algebra systems (such as Magma and SAGE) patterns were discovered from the output of various domination experiments, some of which led to conjectures and eventually to theorems. One computer generated example led to a whole new theory of efficient domination in Cartesian products of graphs.
Fractals with Google Sketchup B. Sidney Smith, Radford University CP5	Google Sketch-up is a free, intuitive, and powerful computer-aided drawing (CAD) program with features that make it an excellent tool both for creating and for teaching about fractals. This talk will introduce the software and demonstrate the techniques of rendering both 2 and 3-dimensional models of fractals, including Sierpienski triangles and Menger sponges.
A Mathematical Consideration of the Single Pendulum James Sochacki, James Madison University CP2	The differential equation (ODE) governing the motion of a mass attached to the end of a rod of fixed length whose other end is attached to a fixed swivel that only allows planar motion can be determined from Newton's ideas. This physical system usually called the motion of the single pendulum is still often studied and modeled by physicists. In this talk I will consider the mathematics of the ODE used by the physicists and perform polynomial projective operations on this ODE that lead to new ODE's. I will then consider what these ODE's tell us about the physics of the single pendulum. I will use recent theories discovered about polynomial systems of ODE's. This talk can be used as a project to give in dynamical systems courses.

A Brief History of Circle Packing Eve Torrence, Randolph-Macon College CP5	An optimal circle packing is an arrangement of circles within a boundary, usually another circle or a square, which maximizes the ratio of the area of the circles to the area enclosed by the boundary. This is a very young field with the case for 10 equal circles verified in only 1997. We will discuss the short yet intriguing history of circle packing and the currently known results.
Simple Ciphers for Small Secrets William P. Wardlaw, retired CP4	Have you ever forgotten your combination? Your PIN number or an account number? Or would you like to send a brief private e-mail? This paper suggests solutions to all of these problems.

Contributed Paper Student Abstracts

On minimizing users' cost in a public transit system Allison Chelsea & Hughes Arthur, Mount St. Mary's University Faculty Advisor: Luca Petrelli CP2	The instability of the economy and the unavailability of natural resources are causing companies to reevaluate their business practices. In doing so they are utilizing methods to minimize cost and not sacrifice customer satisfaction. With the help of Graph Theory we will create an algorithm designed to visit locations with high customer demand to create an efficient transportation system.
Retracts in Category Theory William Ella, University of Mary Washington Faculty Advisor: Randall Helmstutler CP2	We first discuss retract functions in topology and give various examples in different types of spaces. We then explain many of their properties and characterize special types of known retracts. Next we develop a background in the basics of category theory in order to generalize retracts to the categorical level of abstraction. Finally, we apply our general retract definition to other common categories and explore the differences between them.
Designing Codes to Fit Your Needs: An Investigation Into the Construction of BCH Codes Jacob Farinholt, University of Mary Washington Faculty Advisor: Keith Mellinger CP3	In this paper we look at the construction of a particular subclass of cyclic codes known as the Bose-Chaudhuri-Hocquenghem (BCH) Codes. These codes are constructed with a prescribed minimum distance, which means that the codes can be designed to correct as many errors as are required for the intended application. Our goal is to construct classes of BCH codes in as simple a fashion as possible, and we explain what we mean by this. When we desire our codes to correct a particular number of errors, we look at the corresponding polynomial ring and its ideals in order to determine algebraic conditions that would lead to the desired properties. This leads us to number theoretic arguments involving powers of elements in certain finite fields. The results involve the construction of BCH codes with small sets of generators.
Mathematical Modeling of Non-linear Amplifiers Robert Hembreer, Hampden-Sydney College Faculty Advisor: Marcus Pendergrass CP2	In this talk we explore the mathematical modeling of a non-linear amplifier. Both arctangents and hyperbolic tangents are used to model the nonlinear gain of the amplifier. We then explore their effect through Fourier analysis and power series representations. We also see how the Fourier series may emerge from a power series.

The weight enumerator for a class of LDPC codes defined by hyperovals Katie L. Hunsberger, University of Mary Washington Faculty Advisor: Keith Mellinger CP4	In this paper we look at a class of a low-density parity-check codes that are defined by a hyperoval in a finite projective plane. The construction relies on the geometry of hyperovals, and properties of the codes have been explored previously. In the present work, we look at the weight enumerator of the codes. Our goal is to determine the number of codewords of various weights and use the geometry of the plane to represent the small weight codewords. Our construction of small weight codewords makes use of intersecting hyperovals, and so we investigate the necessary algebraic conditions for a pair of hyperovals to share a specified number of points. We determine that all codewords in the projective plane $PG(2, 4)$ arise from the geometric configurations we enumerate. For larger values of $q = 2^s$, our investigations show that for $PG(2, 2^s)$, the parity of s affects the maximum number of points in which a pair of hyperovals can intersect, as well as the total number of hyperovals that intersect in a given number of points. As a result, the parity of s affects the number of small weight codewords resulting from these geometric configurations.
F Chomp: A Variation of Chomp? Andrew MacLaughlin, St. Mary's College of Maryland Faculty Advisor: Alex Meadows CP3	The combinatorial game Chomp, invented in 1974 by David Gale, remains a highly researched topic. Shortly after its creation, Chomp was found to be equivalent to an older combinatorial game called the Divisor Game, created by Fred Schuh. The Divisor Game is played by two people taking turns naming divisors of a single starting number with the stipulation that no number can be a multiple of an already stated number, with the loser being the person who names 1. We generalize the Divisor Game to have multiple starting numbers, with players naming numbers that are divisors of at least one starting number. All other rules of the Divisor Game would still apply. This generalized version of the Divisor Game has several deep connections to both the Divisor Game and Chomp.
Improved Covariance Eigenvalue Estimates and Line Estimation Jonathan Stallings, University of Mary Washington Faculty Advisor: Debra L. Hydorn CP5	Geographic information systems (GIS) involve certain types of vector data that can be used to estimate true points on Earth and a line between two points. Assuming a vector of longitude (X) and latitude (Y) data has a bivariate Normal distribution, we can create $100(1 - \alpha)\%$ confidence regions for the true location. Researching different methods of estimating covariance matrices allows us to create different confidence regions and compare which are better estimates in certain situations. Taking vector data from two different points, we can also create an estimate of the true line between the two points from the initial vector data. This paper focuses on the construction of line estimates and attempts to improve upon them using the different methods of estimating covariance matrices.

Pseudospherical Cone Points Brian Tennyson, St Mary's College of Maryland Faculty Advisor: Ivan Sterling CP5	We study pseudospherical surfaces (or ps-surfaces). That is surfaces with constant Gaussian curvature K=-1. In particular we study ps-graphs which have a conical singularity at the origin. We seek necessary conditions for the existence of examples and to also find new examples.
Combinatorics of Waterfront Compositions David Weirich, Virginia Wesleyan College Faculty Advisor: Dante Manna CP3	Inspired by the shapes of building complexes that are subject to height restriction laws, we define a new kind of restricted composition number and study its properties.
A non-elliptical introduction to elliptic curve cryptography Andrew Wills, Randolph-Macon College Faculty Advisor: Adrian Rice CP4	This talk will introduce some basic properties of elliptic curves and explore how those properties can be used to encode and encrypt messages in a public key setting. These cryptographic systems take advantage of the discrete logarithm problem and depend on the fact that points on an elliptic curve form an abelian group under addition. In particular, an example using an analog of the ElGamal cryptosystem will be shown.
COMAP Presentations	
"Watts" the Deal with American Telephones? Michael DeWitt, Nolan Skochdopole and Aashish Gadani Clover Hill High School COMAP Advisor: Peter Peterson CP5	Over the past two decades, cell phone technology has evolved from the old brick car phones of the early 1990s to the slick media centers of today. The convenience of the cell phone along with its fairly inexpensive operation has caused many American households to drop their traditional landlines altogether. According to a recent survey completed by the Nielsen group, nearly 20 million households, 17.1% of all households in the United States, have gone wireless, cutting the cords in favor of their cells. The goals of this research project were to analyze the energy consequences of this social trend and to model the telecommunications network of a demographically identical nation to the United States, optimizing the system to use a minimal amount of energy.
The Cell Phone Revolution Patrick O'Neil, Evan Menchini, Will Frey Virginia Tech COMAP Advisor: John Rossi CP6	Since the mainstream introduction of the cell phone in the mid 1990s, the vigor with which these phones have permeated all facets of modern life has been astounding. As the number of cell phone users in the United States grows, so too does the energy requirement to keep phones charged and running. Using recent and accurate data, we review the growth of cell phone popularity in the United States and look ahead to predict future popularity and energy needs for this cell phone revolution.
Rules of the Road Around a Roundabout Toby Shearman, Patrick Lafond Virginia Tech COMAP Advisor: Henning S. Mortveit CP4	Compared to more conventional intersection types, modern roundabouts provide a safer and more effective way to cope with increasing traffic flows and urbanization. Roundabouts are gaining global popularity as a result. Predicting the dynamics of an existing roundabout or traffic circle is a challenging task, and consequently, optimization of traffic-control configurations is also challenging. Considering overall traffic distributions and traffic circle size, we developed a generalized model to determine relative efficacies of each

	traffic-control configuration—the placement of yield signs, stop signs, or traffic lights. By approaching traffic circles more generally, our proposed method provides a means to improve existing roundabouts, and to convert historic traffic circles into structures resembling modern roundabouts. Validated against basic simulations of existing roundabouts, our model suggested the most efficient configurations.

Undergraduate Poster Descriptions

Mathematics in India Jillian Dixon, Randolph-Macon College Faculty Advisor: Chirashree Bhattacharya	This poster presents some highlights of students' experiences during a January term mathematics travel course to India.
A Polynomial Analogue of the Collatz Problem and its Fractal Characteristics Brian Kim, Marriotts Ridge High School Faculty Advisor: Kenneth Hicks (Ohio Univ.)	A polynomial analogue of the Collatz problem and its fractal characteristics are explored and compared to a dyadic diagonal Cantor set.
Some New Accurate Approximations to e Brooke Rowe, Christopher Newport University Faculty Advisor: Hongwei Chen	In contrast to the computation of Pi, there are few studies in calculating the natural number e to comparable precision. This is because its well-known Maclaurin series is quite accurate. In this poster, we present some alternative, accurate approximations to e. All approximations are based on closed form expressions and use only elementary calculus.
Audio Compression Using The Fourier Transform Alex Smith, Hampden-Sydney College Faculty Advisor: Marcus Pendergrass	We consider an audio compression technique that uses the Fourier Transform to store only the most important frequency components of an audio signal. An overview of the basic algorithm is given, and a real-time demonstration of the technique will be given using a laptop computer.