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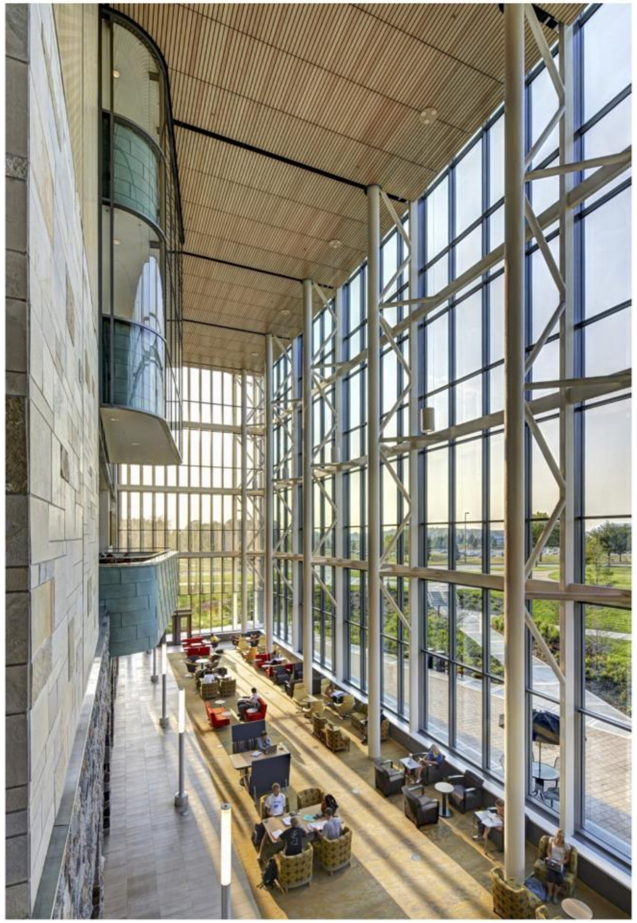
My IBL Approach to Theoretical Ideas in First-Semester Calculus

Context

- "Most selective" liberal arts college
- ~1750 undergraduates
- At most 24 students per section
- Four meetings per week for 12 weeks
- Calculus for students who've had calculus before
  - Limits, derivatives, applications of derivatives, FTC, integration by substitution
- Flipped class
- Feedback and assessment

ACTIVE CALCULUS

2016 Edition



Matthew Boelkins

David Austin Steven Schlicker

3.4.1 Derivatives of the cotangent, secant, and cosecant functions

Subsection 3.4.1 Derivatives of the cotangent, secant, and cosecant functions

In Preview Activity 3.4.1, we found that the derivative of the tangent function can be expressed in several ways, but most simply in terms of the secant function. Next, we develop the derivative of the cotangent function.

Let $g(x) = \cot(x)$. To find $g'(x)$, we observe that $g(x) = \frac{\cos(x)}{\sin(x)}$ and apply the quotient rule. Hence

$$g'(x) = \frac{\sin(x)(-\sin(x)) - \cos(x)\cos(x)}{\sin^2(x)} = -\frac{\sin^2(x) + \cos^2(x)}{\sin^2(x)}$$

By the Fundamental Trigonometric Identity, we see that $g'(x) = -\frac{1}{\sin^2(x)}$; recalling that $\csc(x) = \frac{1}{\sin(x)}$, it follows that we can most simply express g' by the rule

$$g'(x) = -\csc^2(x).$$

Note that neither g nor g' is defined when $\sin(x) = 0$, which occurs at every integer multiple of π. Hence we have the following rule.

Cotangent Function: For all real numbers x such that $x \neq k\pi$, where $k = 0, \pm 1, \pm 2, \ldots$,

$$\frac{d}{dx}[\cot(x)] = -\csc^2(x).$$

Observe that the shortcut rule for the cotangent function is very similar to the

Preview Activity 3.4.1. Consider the function $f(x) = \tan(x)$, and remember that

$$\tan(x) = \frac{\sin(x)}{\cos(x)}.$$

1. What is the domain of f?
2. Use the quotient rule to show that one expression for $f'(x)$ is

$$f'(x) = \frac{\cos(x)\cos(x) + \sin(x)\sin(x)}{\cos^2(x)}.$$

3. What is the Fundamental Trigonometric Identity? How can this identity be used to find a simpler form for $f'(x)$?
4. Recall that $\sec(x) = \frac{1}{\cos(x)}$. How can we express $f'(x)$ in terms of the secant function?
5. For what values of x is $f'(x)$ defined? How does this set compare to the domain of f?

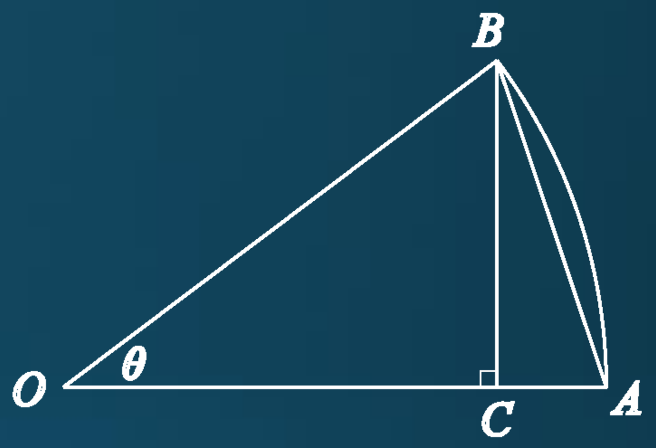
3.4.1 Derivatives of the cotangent, secant, and cosecant functions

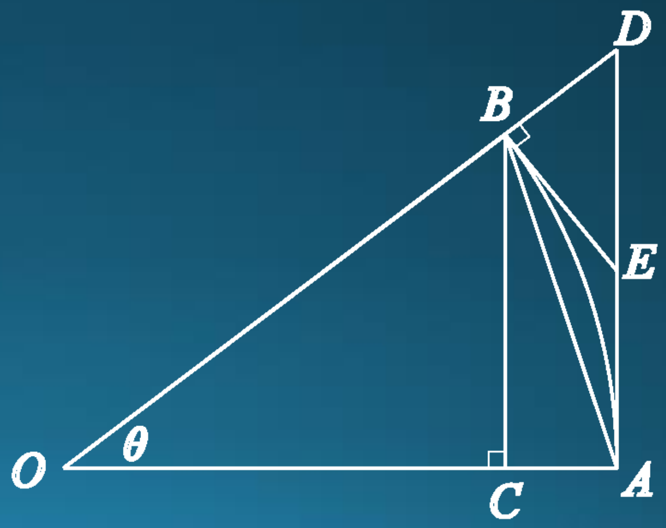
“Missing” content

- Squeeze theorem for $\lim_{x \to 0} \frac{\sin(x)}{x}$
- Intermediate Value Theorem
- Mean Value Theorem
- Natural logarithm as integral function
- Meaningful discussion of why FTC is true

$$\lim_{x\to 0}\frac{\sin(x)}{x}$$

- Squeeze theorem screencast
- In-class IBL activity for full period
  - Motivate from $\frac{d}{dx}\sin(x)$
  - Lots of step-by-step questions
  - Emphasize importance of radian measure
  - High student engagement





Mean Value Theorem

- Assume Rolle’s Theorem
- “Trick” them into proving the MVT
  - Tie back to writing project
- Bounded derivative implies bounded growth
- $f'(x) = g'(x)$ implies $g(x) = f(x) + C$

Logarithm as Integral Function

- Define $L(x) = \int_1^x \frac{1}{t}\,dt$ for $x > 0$
- Show that *L*(*x*) has all the familiar properties of ln(*x*).
- Use IVT to show *L*(*x*) = 1 for some *x*.

Why the Fundamental Theorem of Calculus works

- Recall the Mean Value Theorem
- Guided through some Riemann sums
  - Uses Mean Value Theorem
- Conclude the part about evaluating definite integrals
- Limit definition of derivative to see the part about differentiating integral functions

Handout Source

- For now: email kellermt@wlu.edu
- In future: Open-source contribution to Active Calculus