Sonobe Origami for enriching understanding of geometric concepts in three dimensions

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St. Mary's College of Maryland
November 7, 2015. MAA Section Meeting
3:15 pm -3:35 pm Goodpaster Hall Room 109

"Programs that take advantage of paper folding to teach mathematics are thriving in many parts of the world," according to the organizers of the MAA origami-themed Contributed Paper Session to be held at the JMM in January 2016. But, K-12 should not be having all the fun! In this talk, I will show some ways of stimulating student engagement using sonobe origami. These activities can be used in General Education mathematics "appreciation" courses or for Non-Euclidean Geometry for mathematics majors. Specifically, the goals are enriching student understanding of surface curvatures and helping them understand the duality of the regular polyhedra using these folded paper objects.

Things to chat about...

- Construction overview and classroom hints
- Duality of regular polyhedra

  (aka Platonic Solids)
- Euler's χ for topological classification
- Angle deficit for topological classification
- Gauss-Bonnet Theorem
- Bonus Puzzle!

Construction Basics

- tinyurl.com/sonobe
- I also have videos on YouTube
- for Gen Ed course- one class period
- students spend about an hour outside of class
- 30 pieces of paper per student
- chirality compatibility is a construction issue
- assembly is a learning experience for students

The Paper I Use in Class:

Roll over image to zoom in

Neenah Astrobrights Note Cube, 3.5 X 3.5 Inches, Assorted, 550 Count (20400)

by Neenah

32 customer reviews

Price: $11.07 Prime | FREE Same-Day

Delivered today for FREE with qualifying orders over $35. Details

In Stock.

Ships from and sold by Amazon.com. Gift-wrap available.

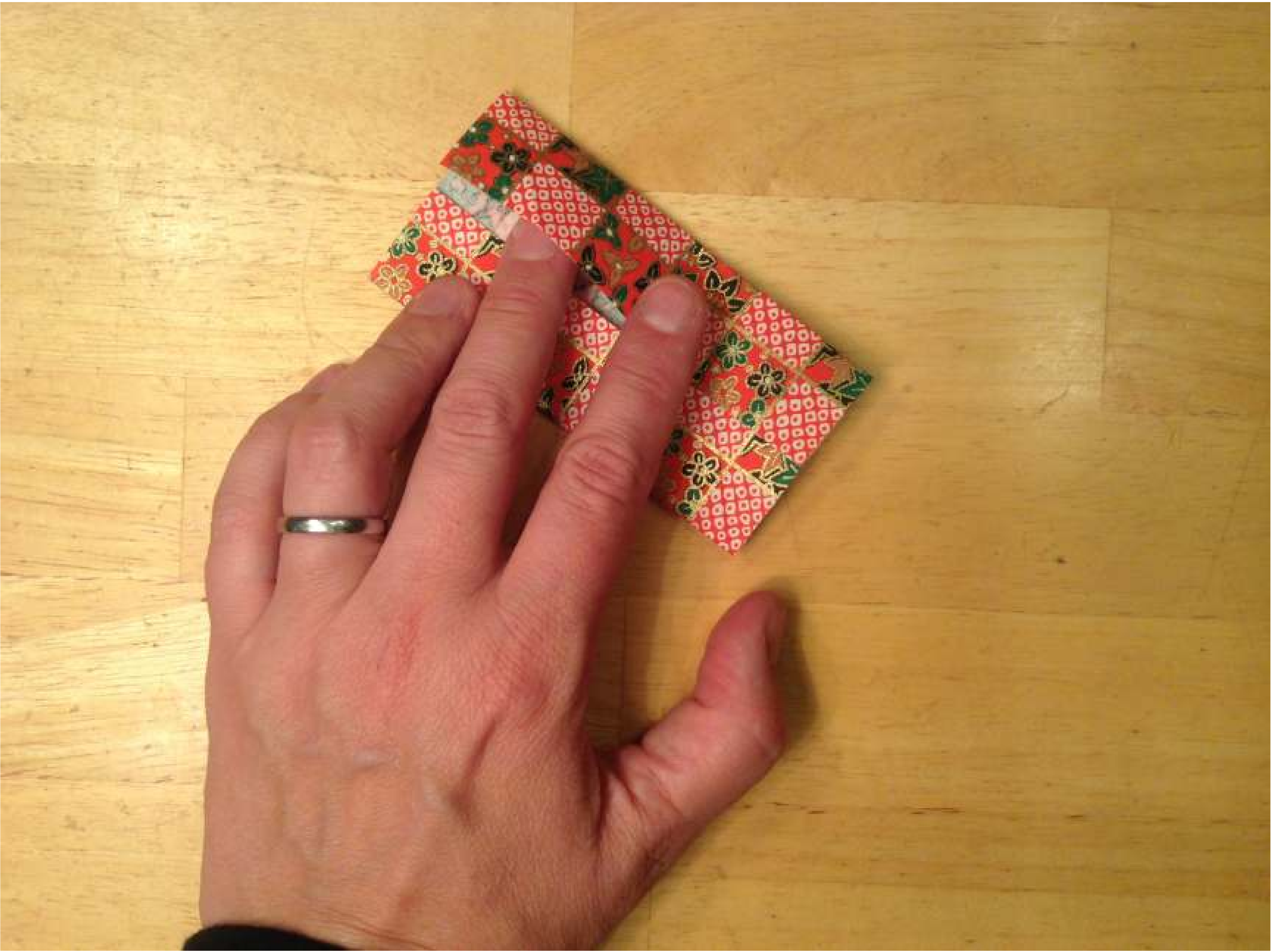
Want it TODAY, Oct. 29? Order within **8 hrs 50 mins** and choose **Same-Day Delivery** at checkout. Details

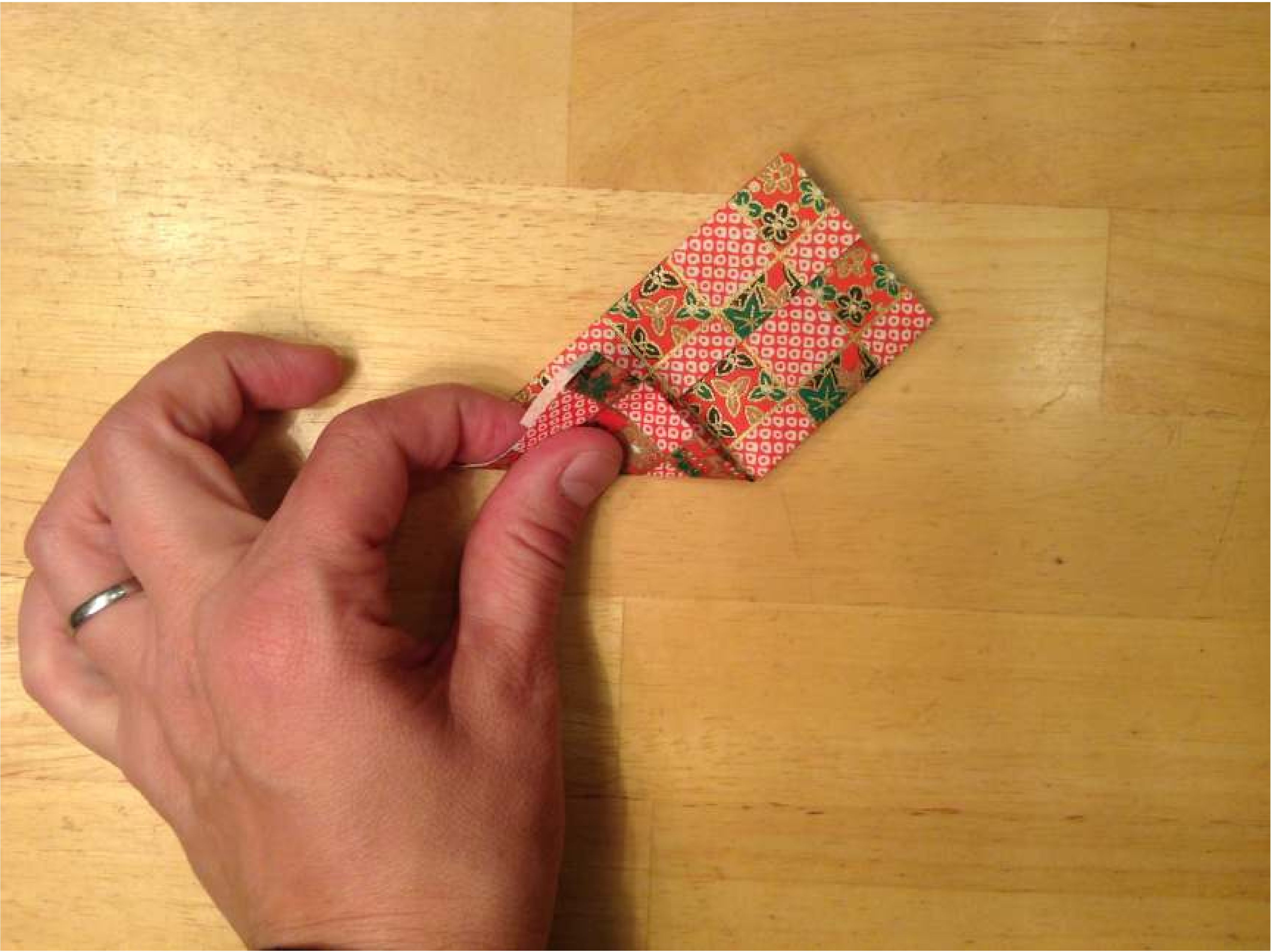
- Acid free for great quality that doesn't deteriorate
- 11 Bright colors
- Perfect for any home or office
- Perfect to keep by the phone
- 550 Sheets

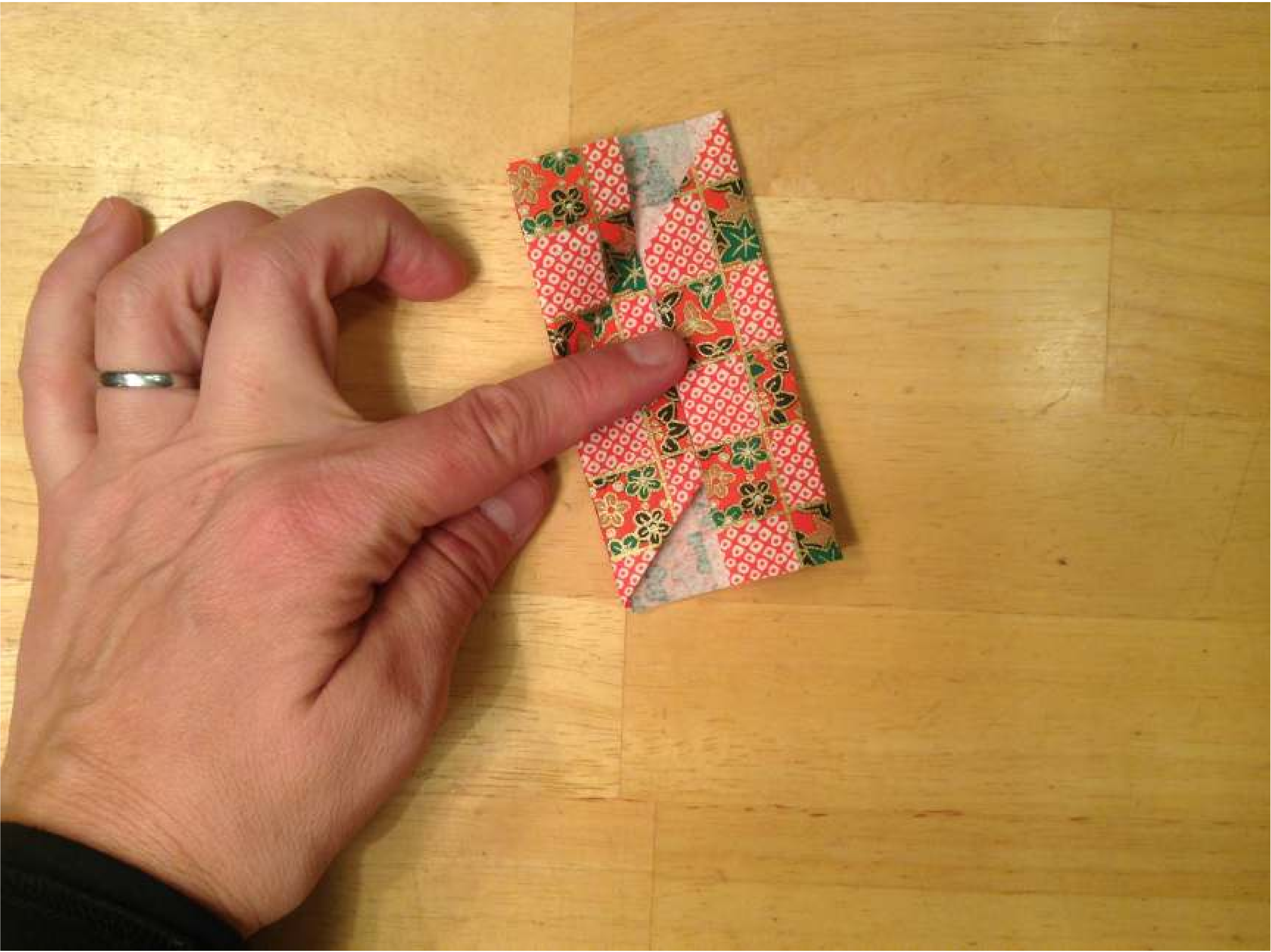
Pros and Cons

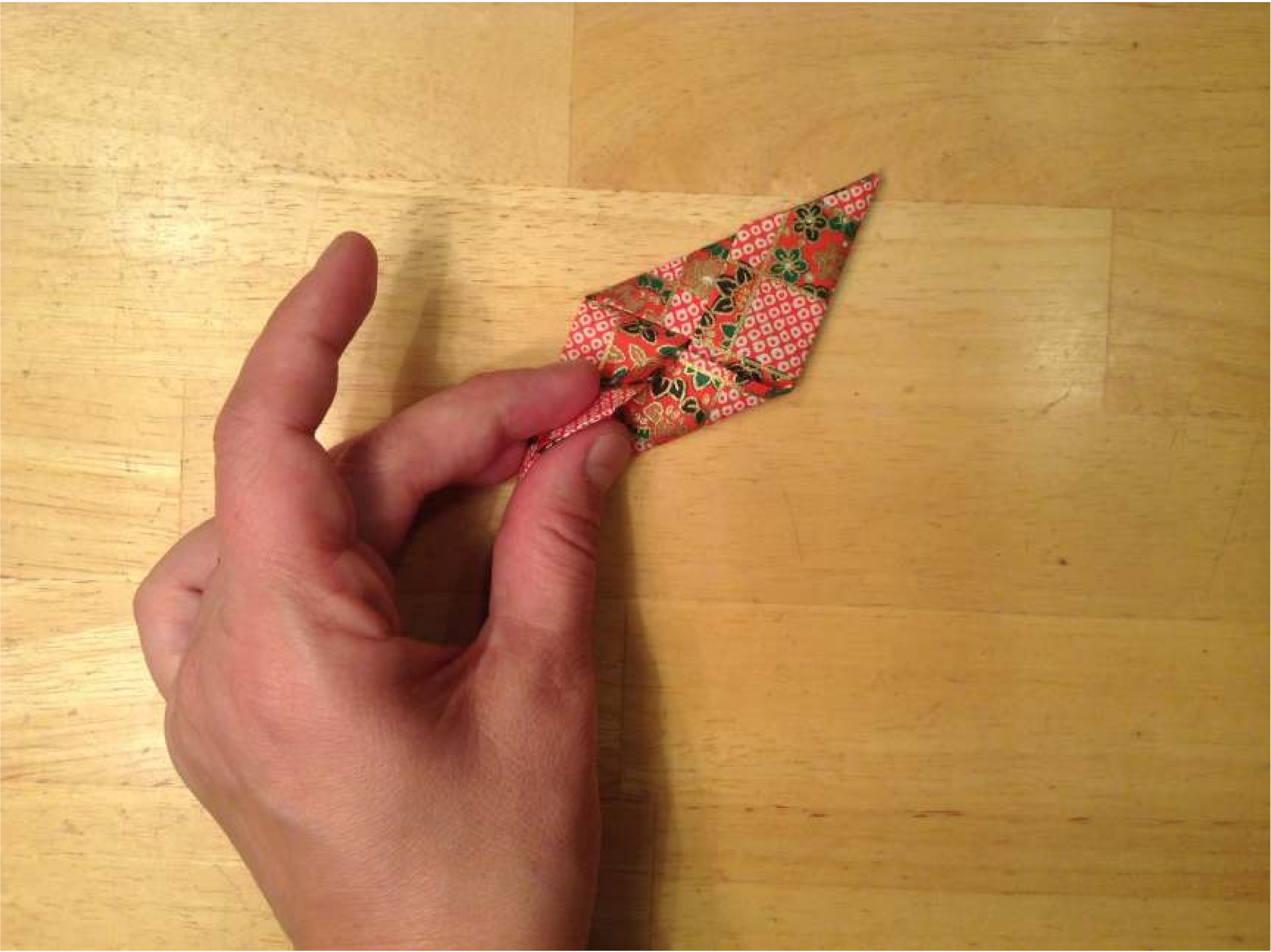
- No glue
- No scissors
- Straightedge and Compass is equivalent to folding
- Pride in crafting

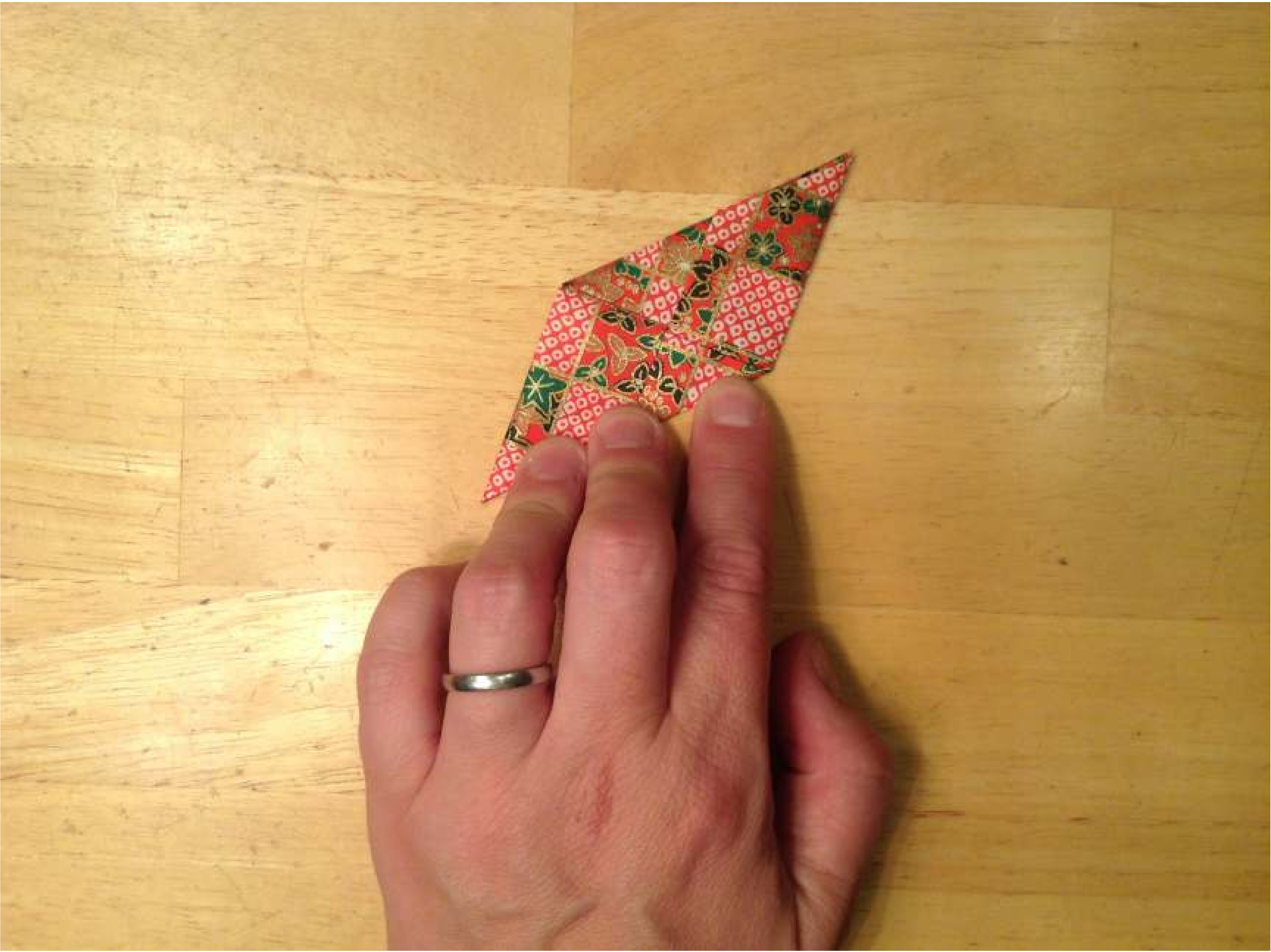
- Only time for one large project rather than many small ones
- Object is more complex than regular polyhedra

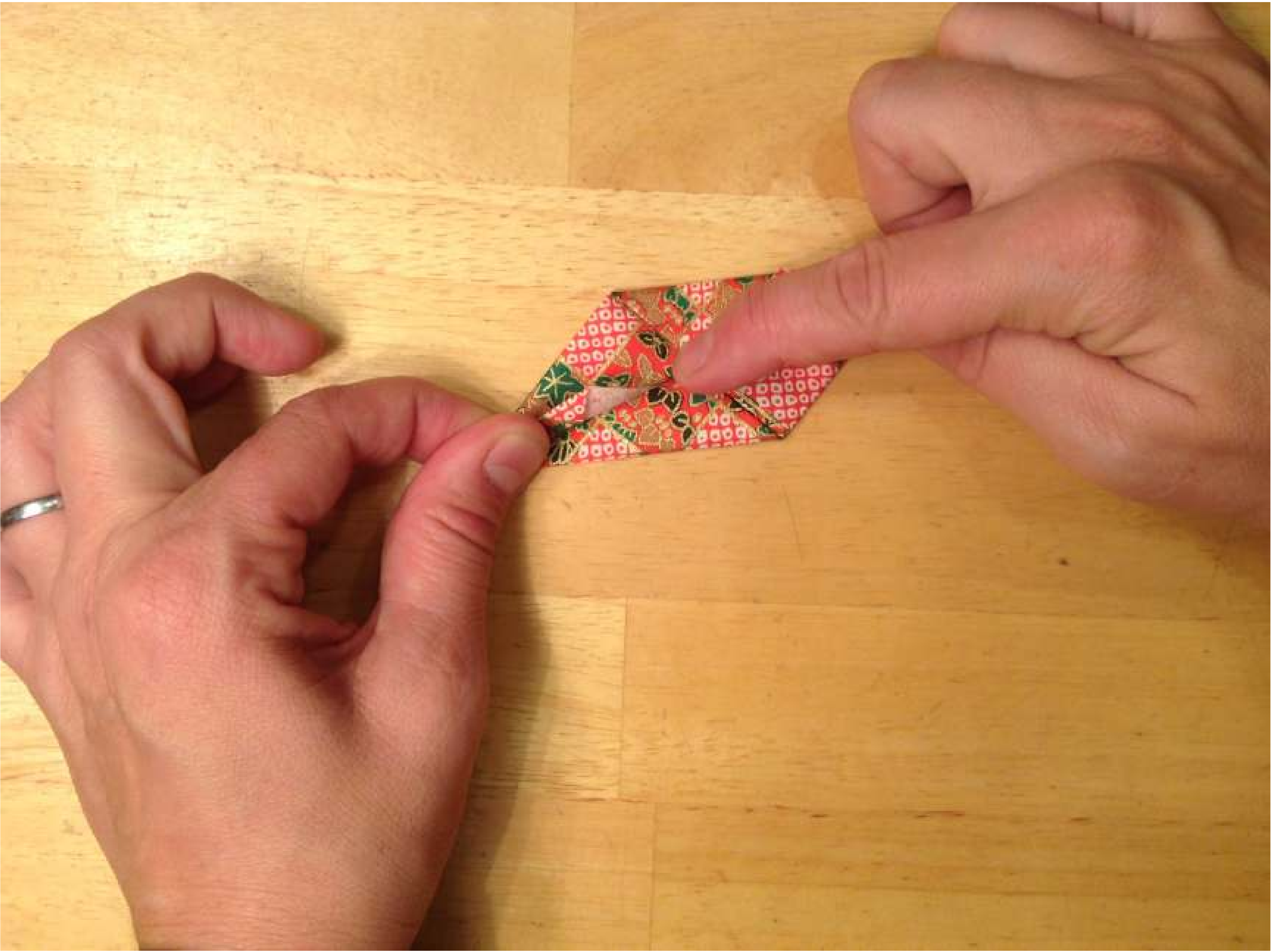


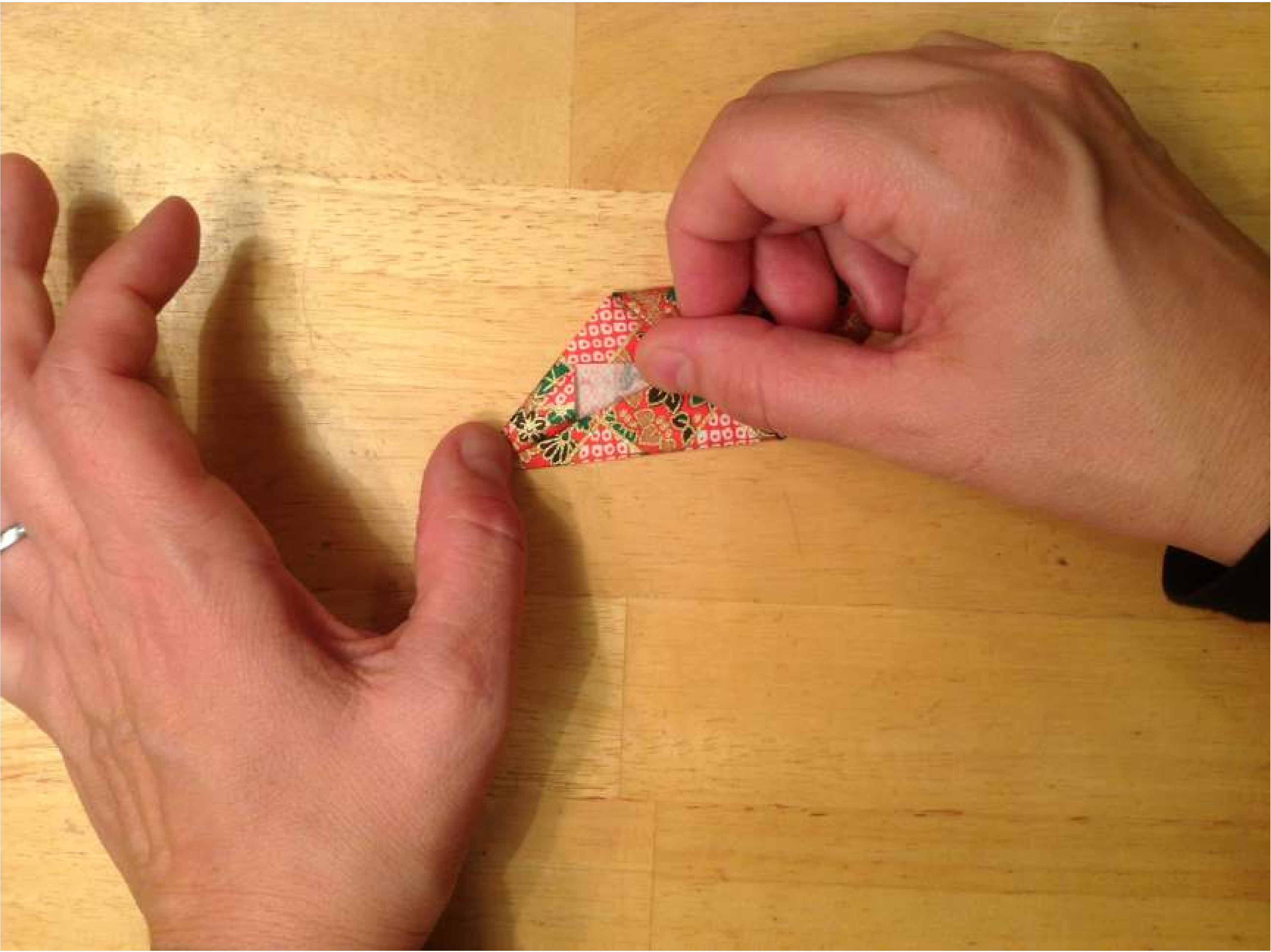


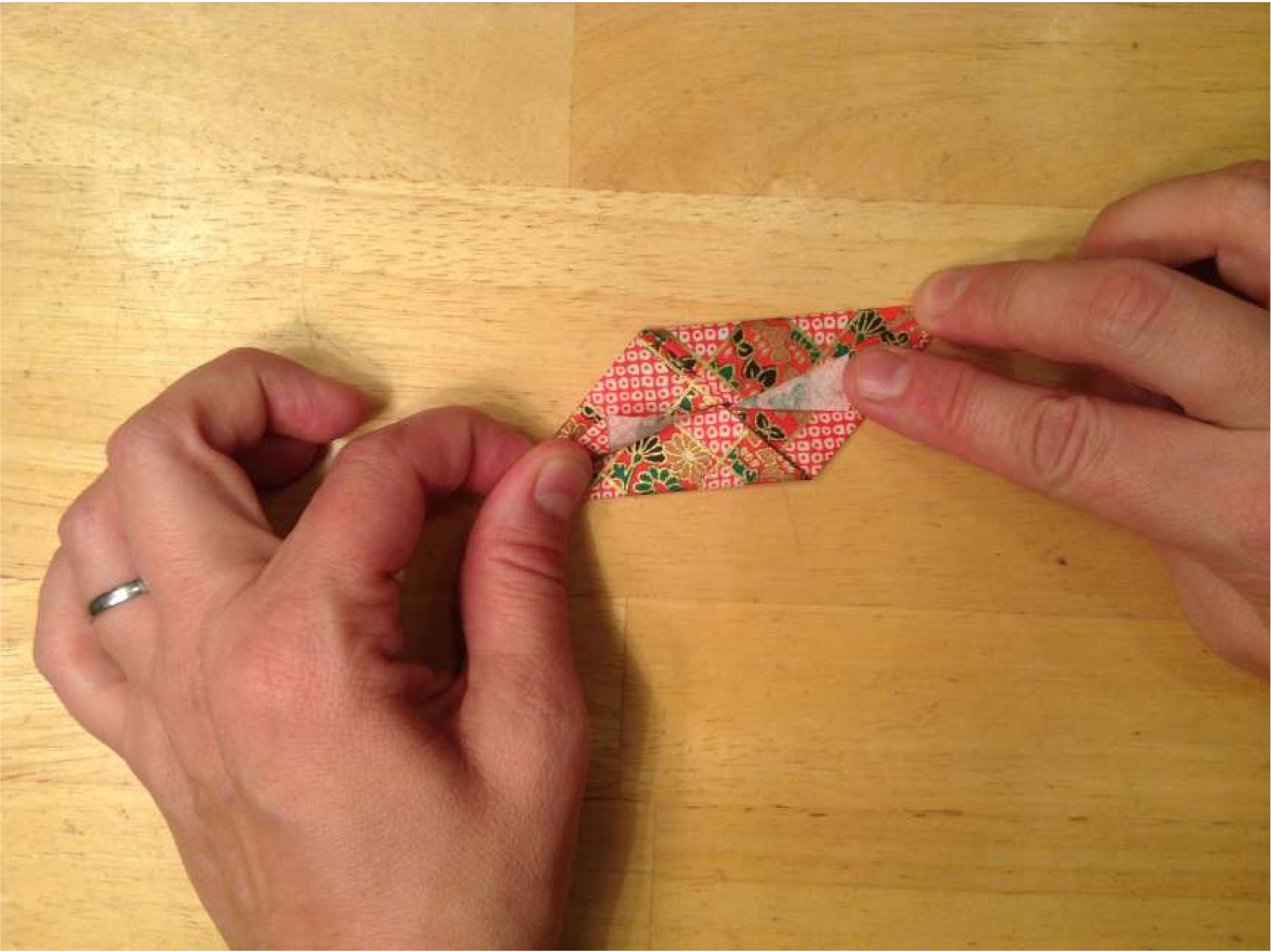


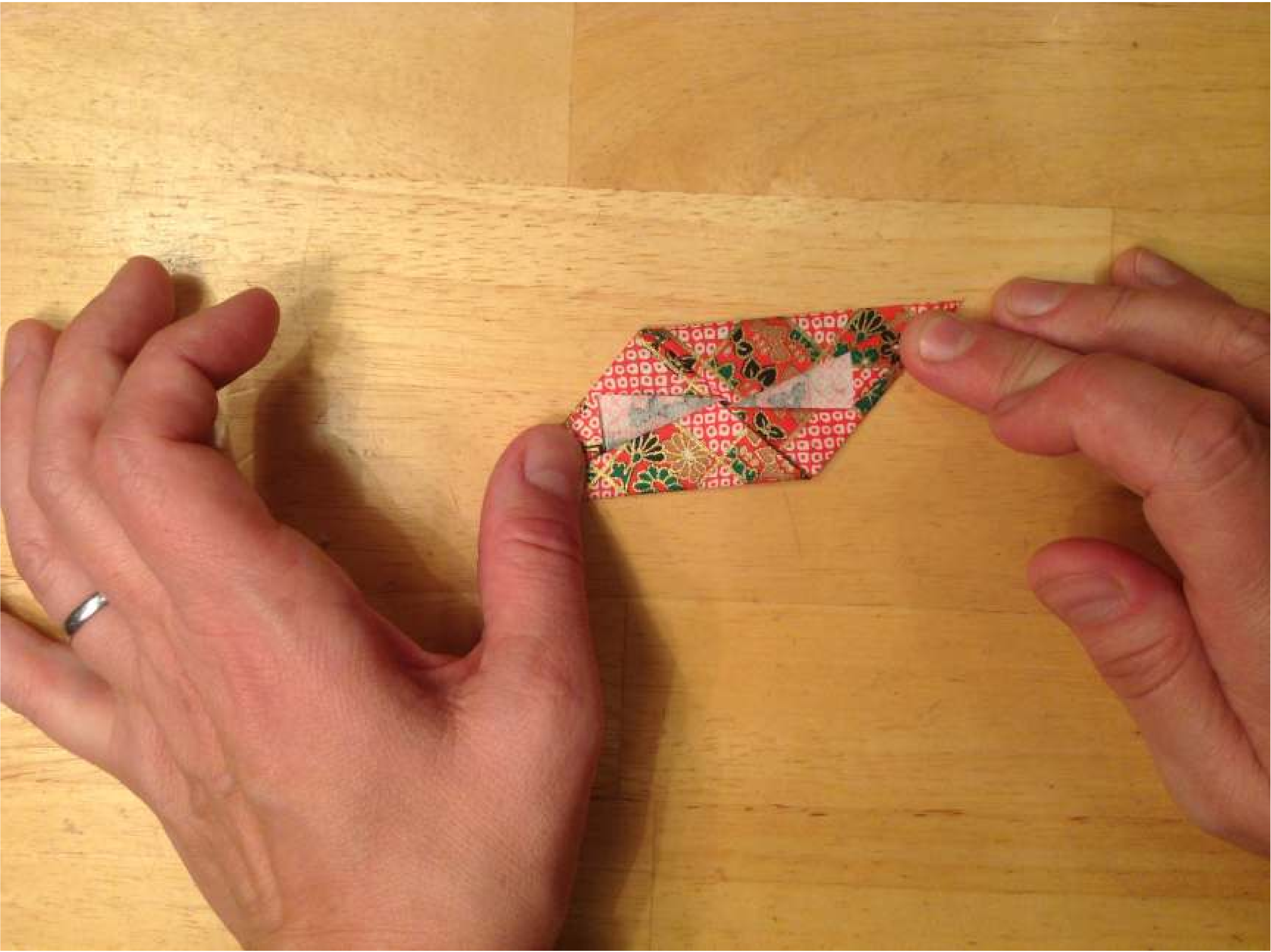


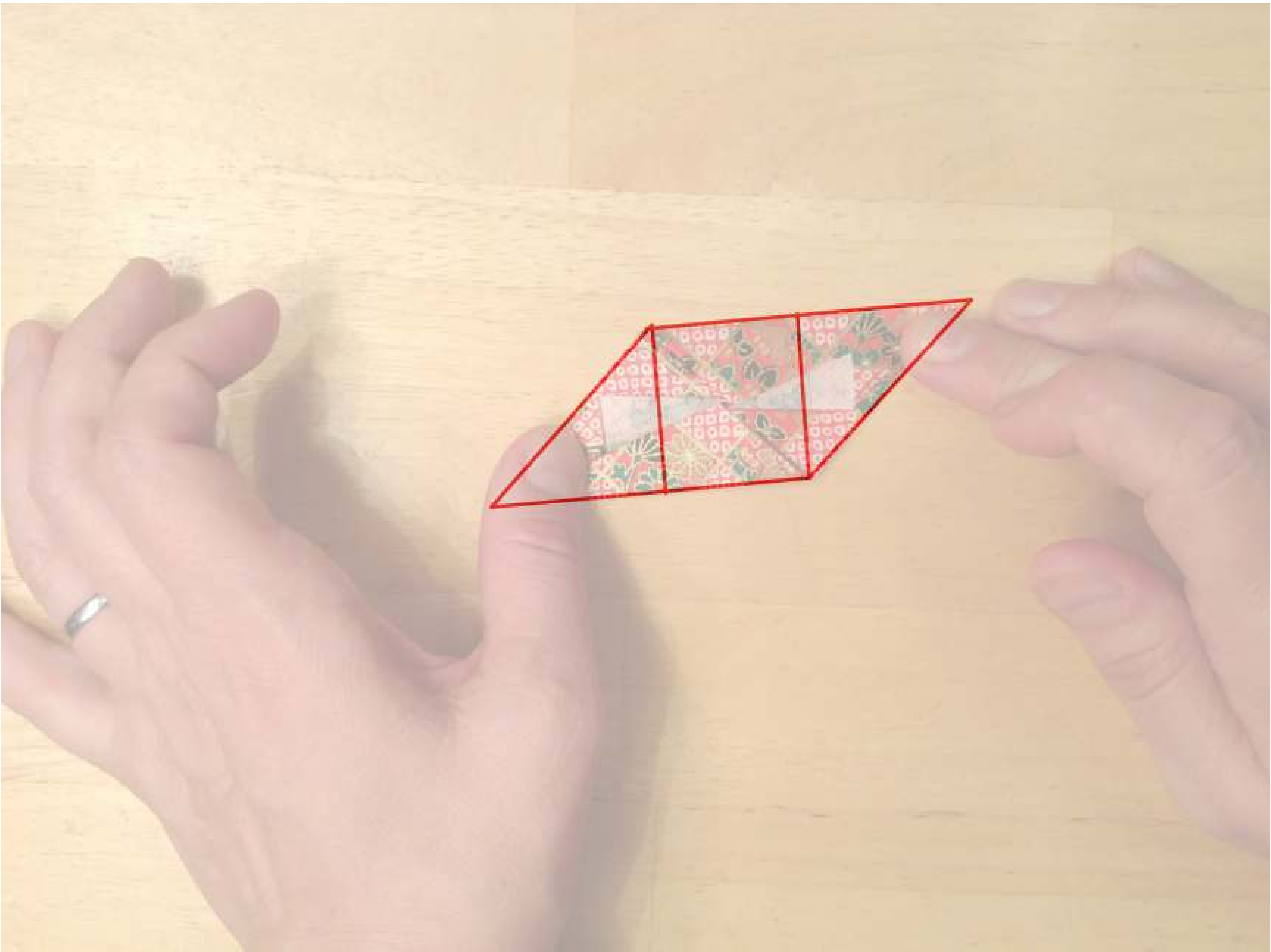


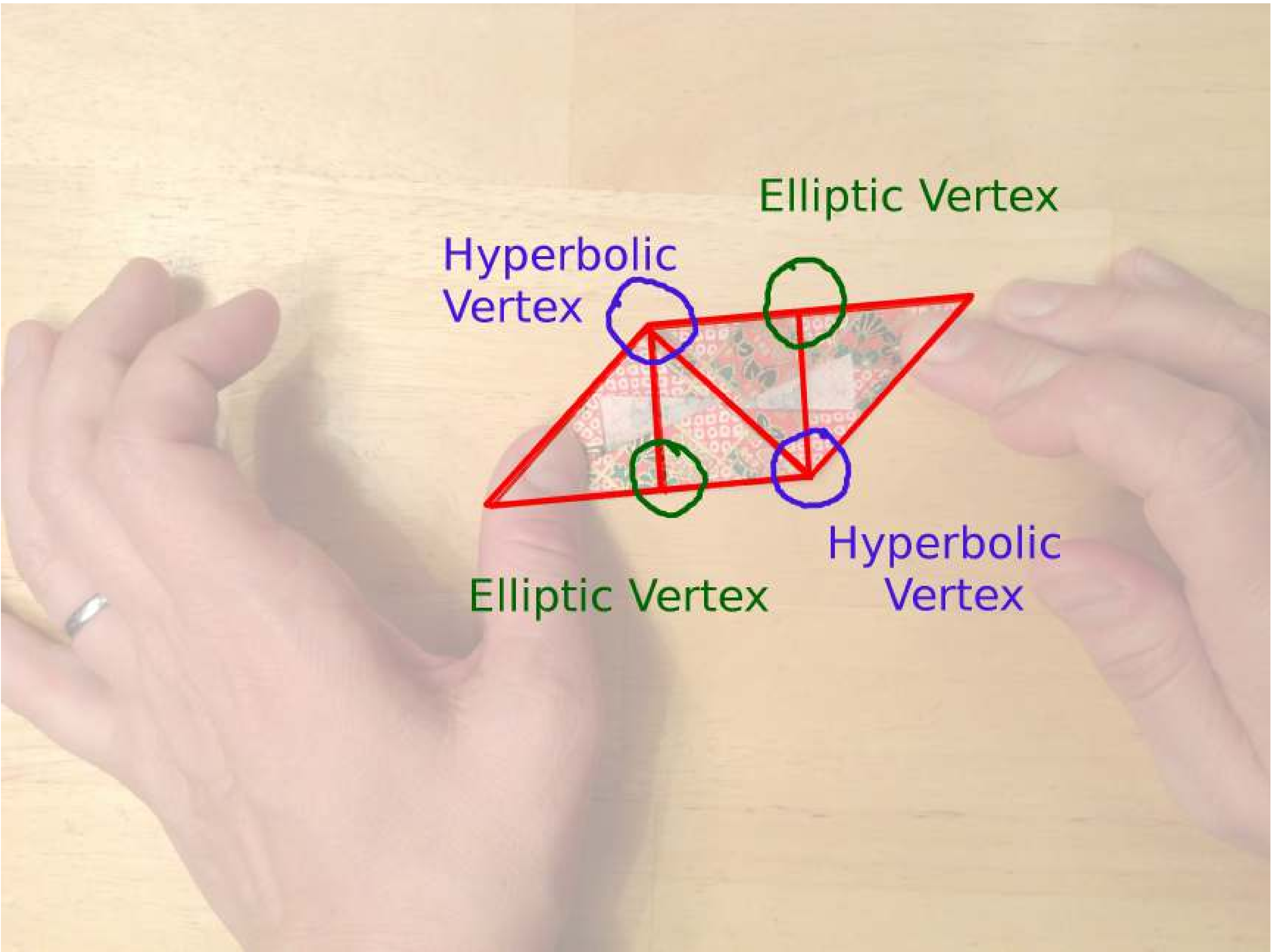


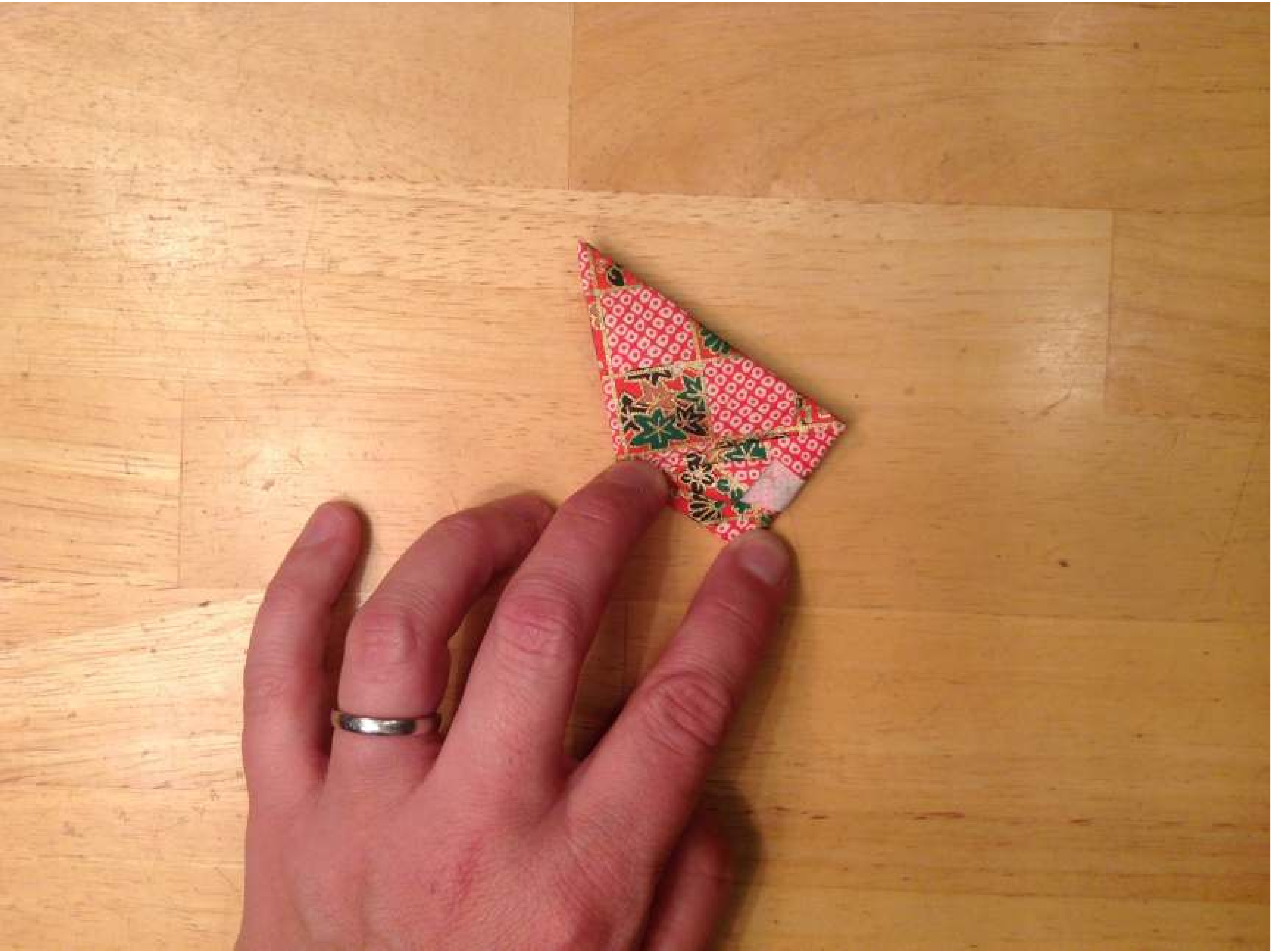




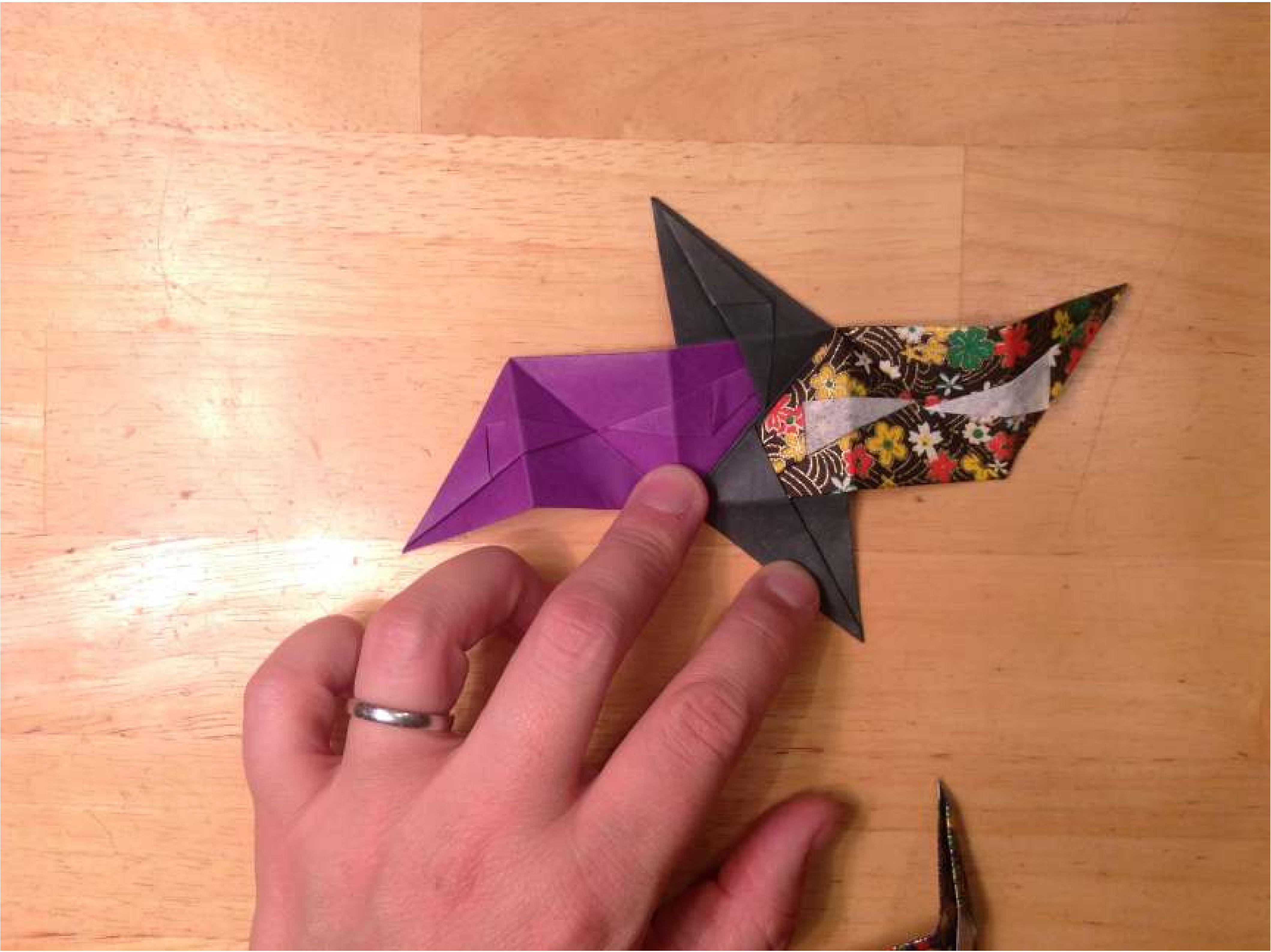


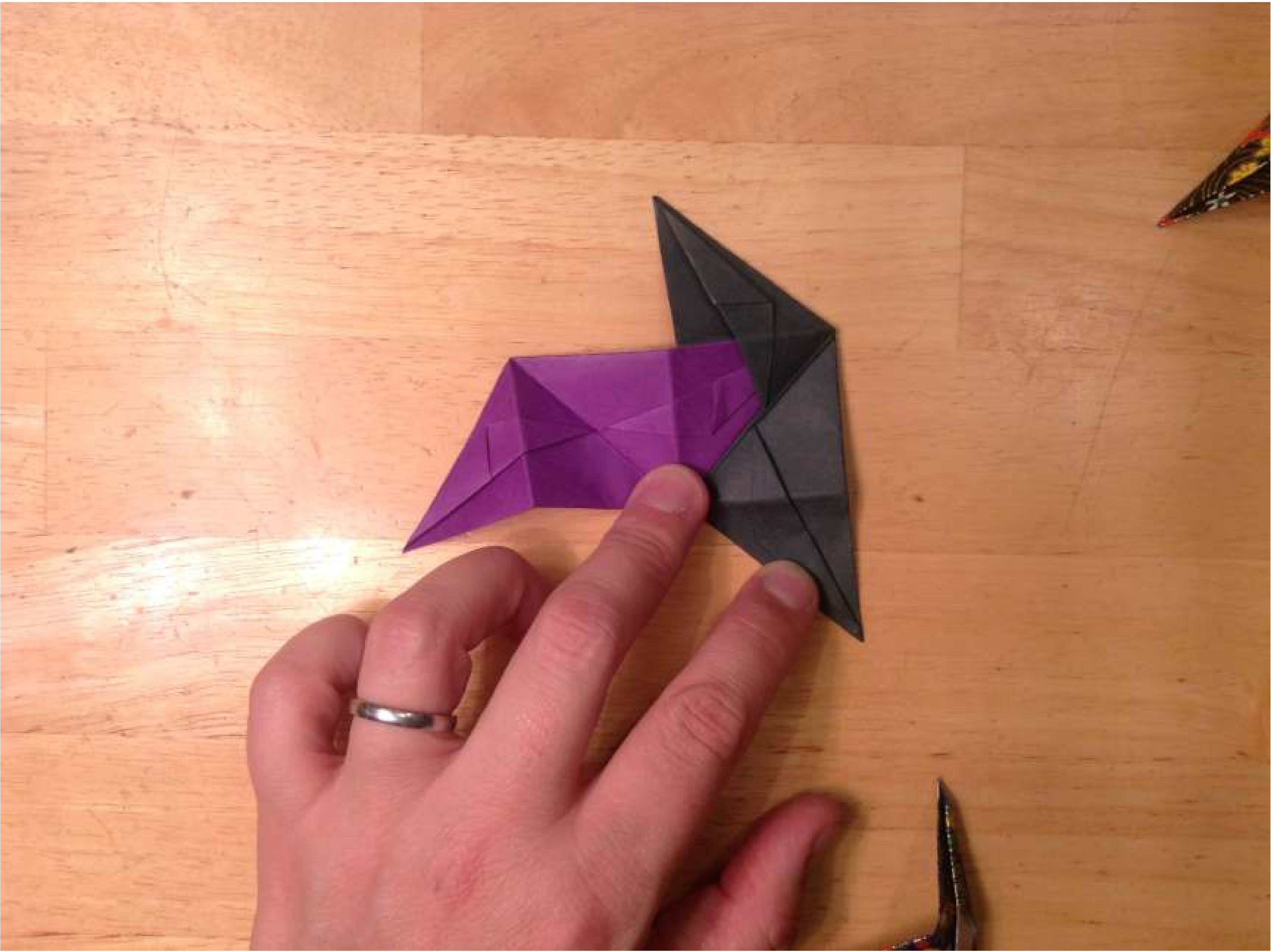


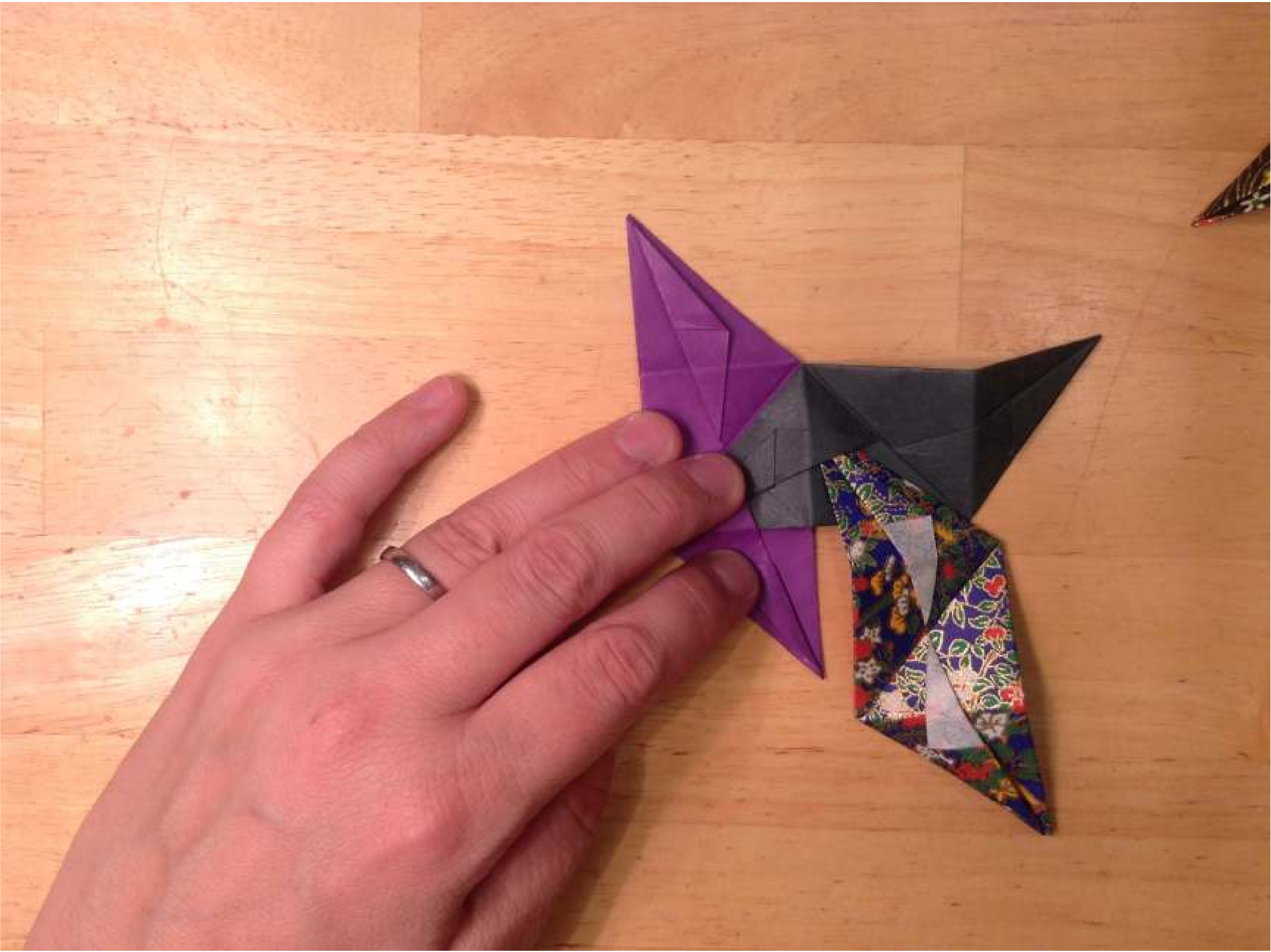

Elliptic Vertex
Hyperbolic Vertex
Hyperbolic Vertex
Elliptic Vertex

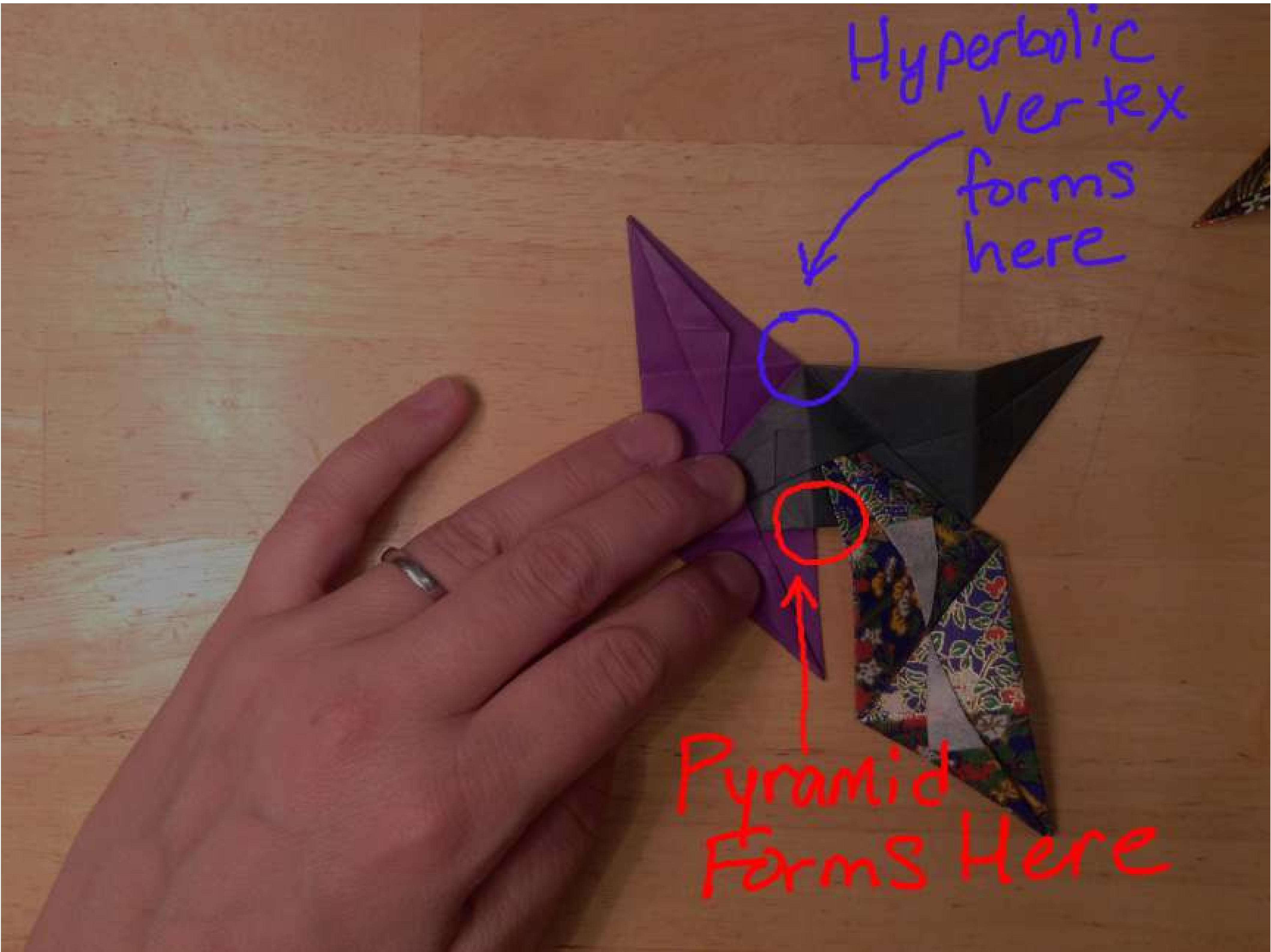


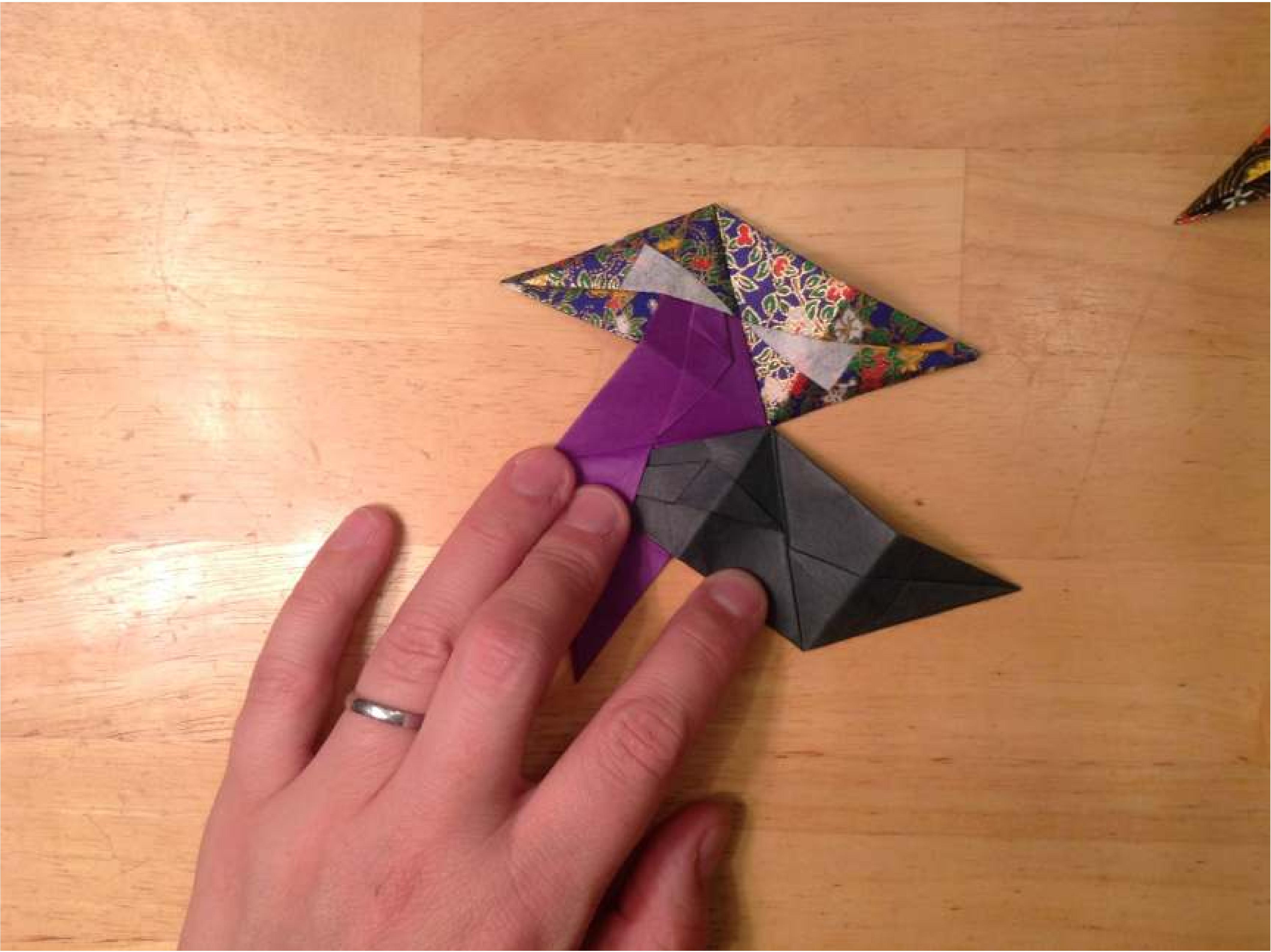
Make 30 of those

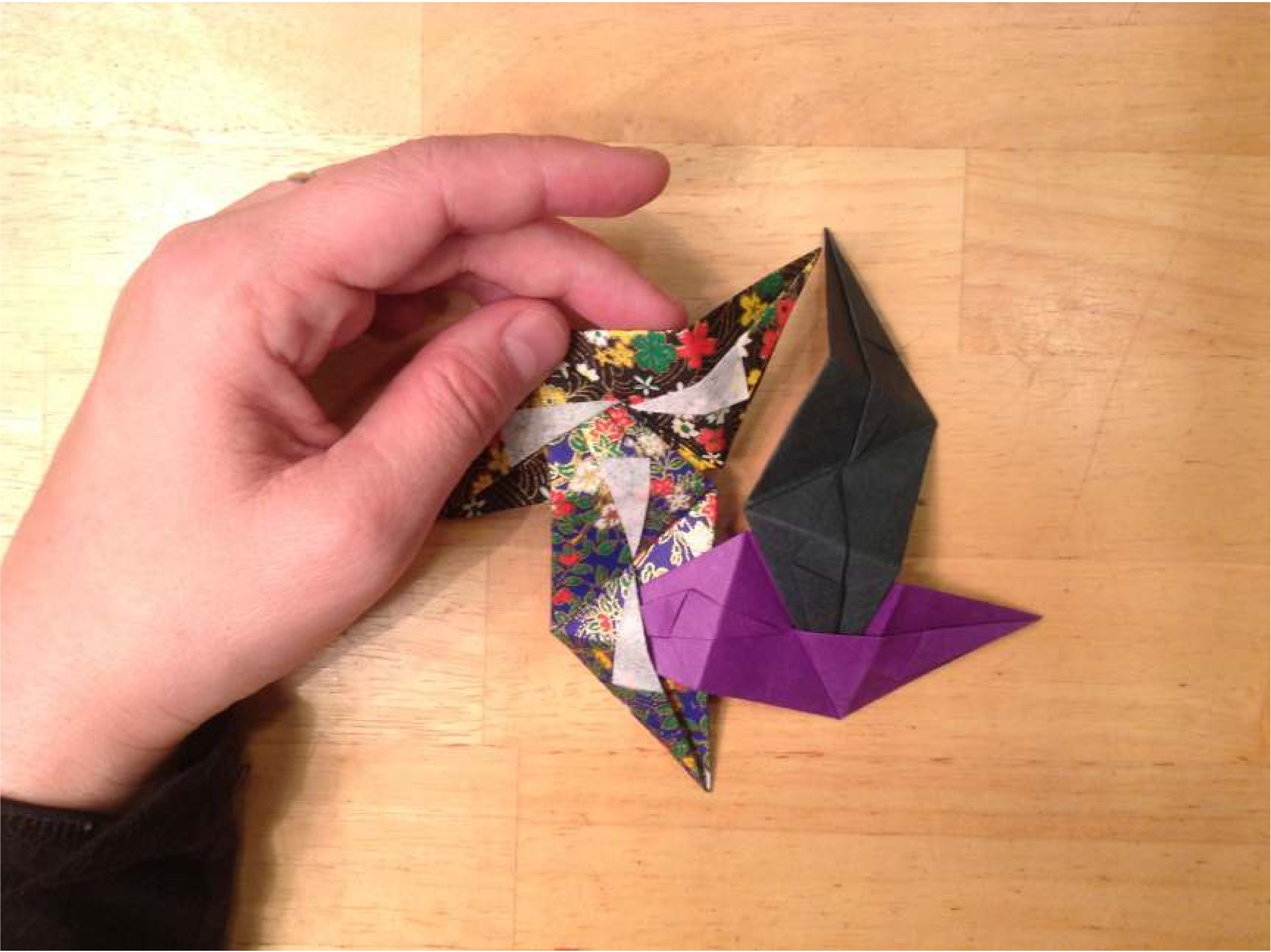


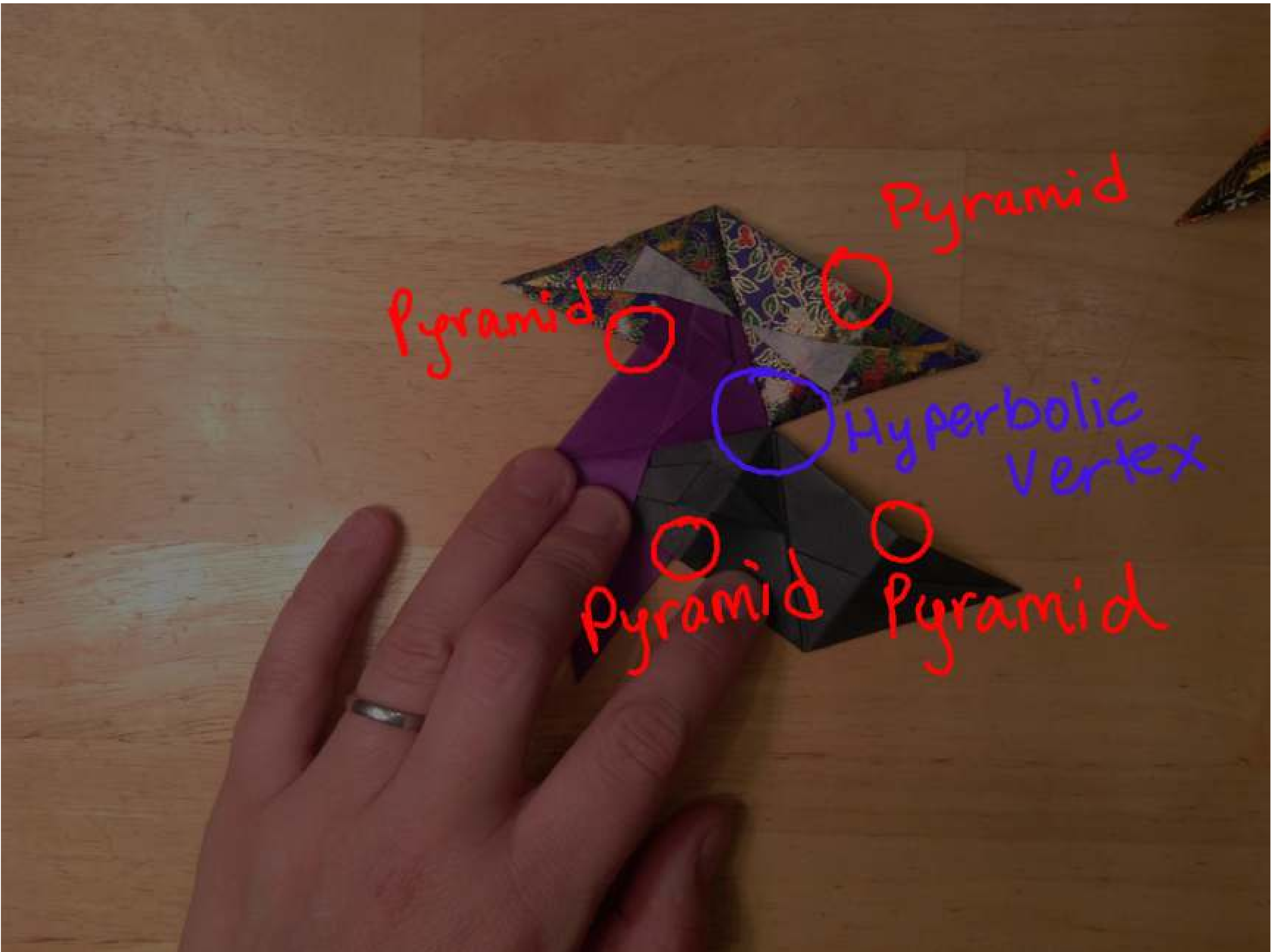


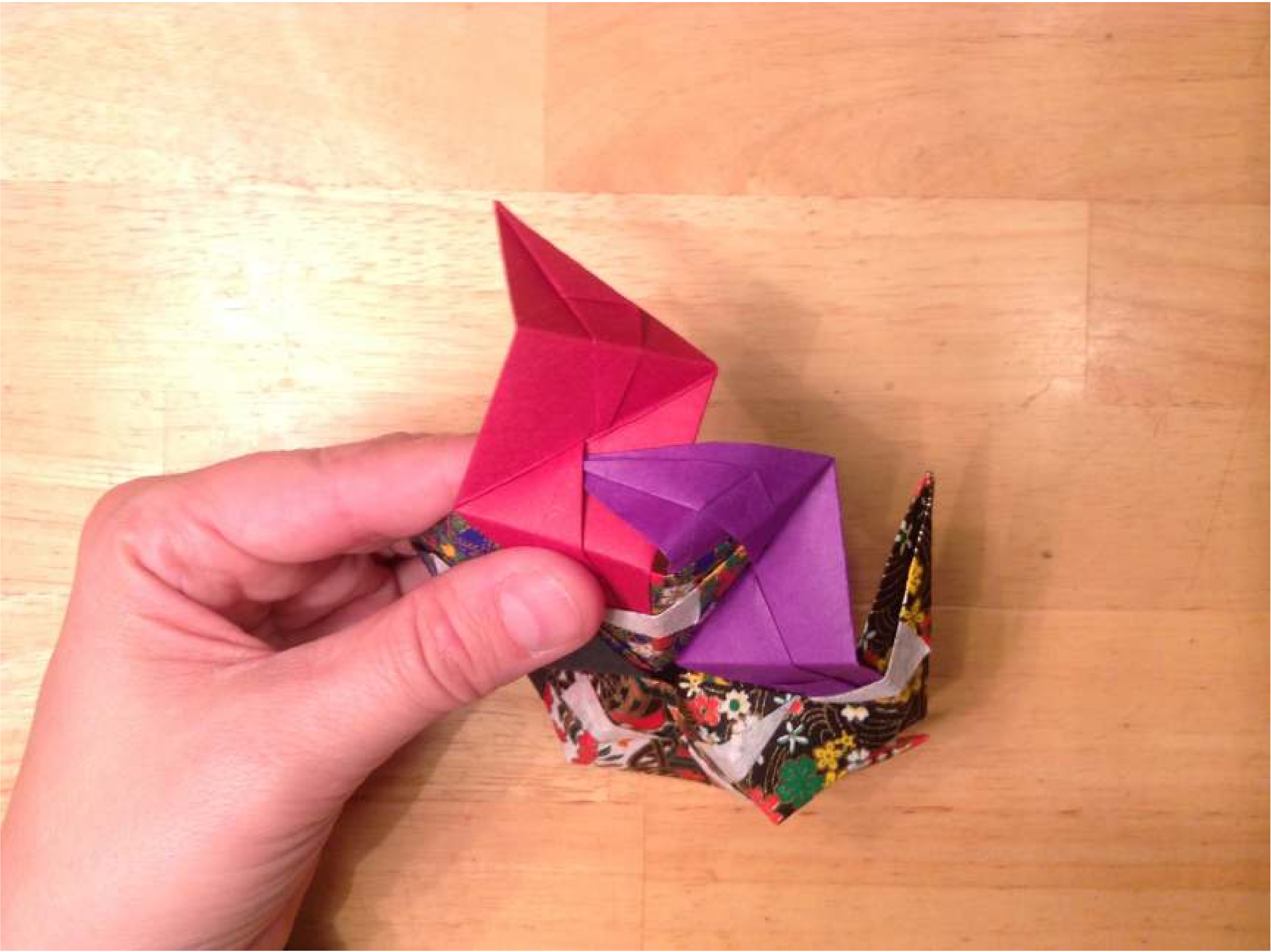


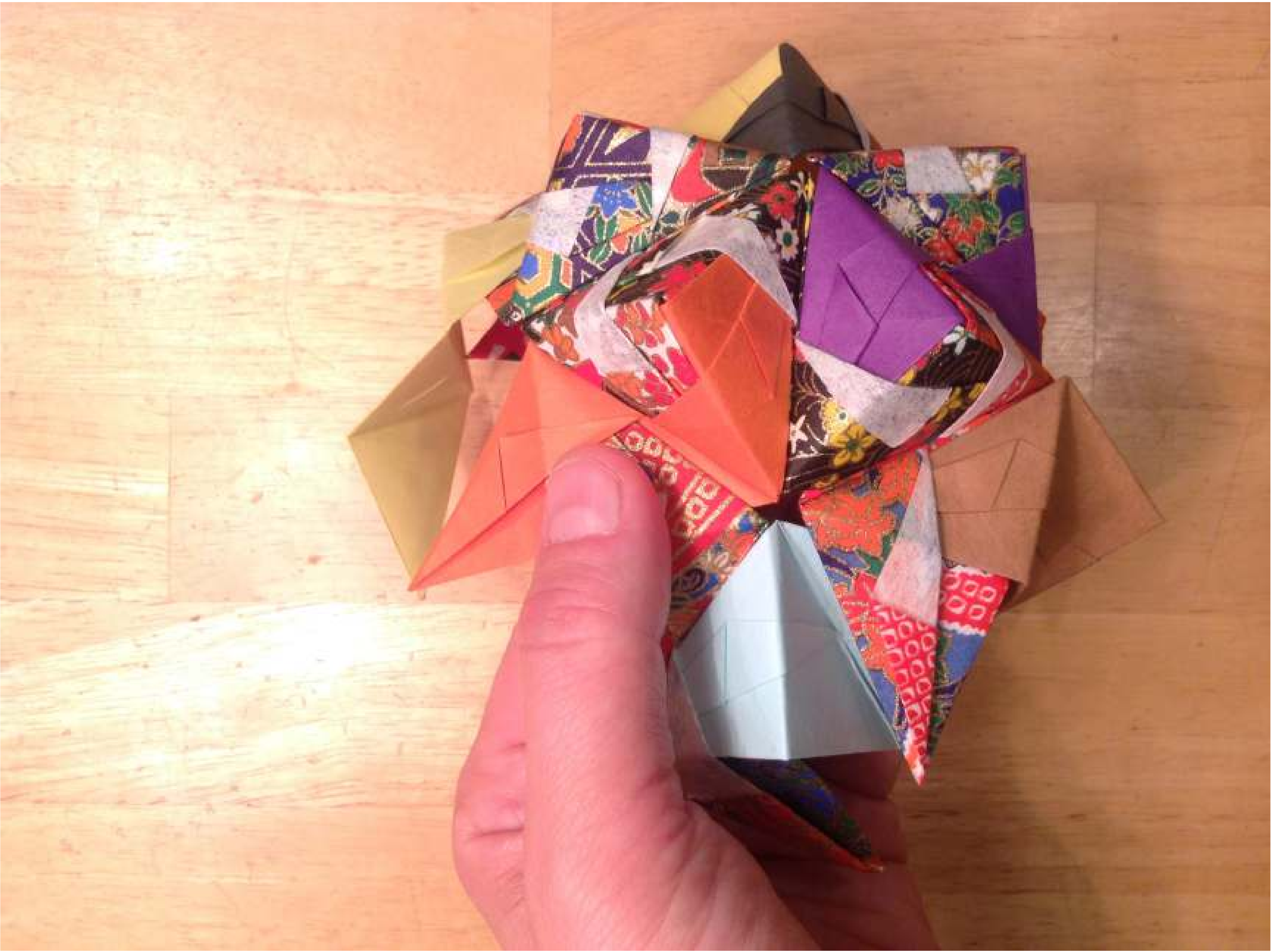

Hyperbolic vertex forms here
Pyramid Forms Here






Pyramid
Pyramid
Hyperbolic Vertex
Pyramid
Pyramid

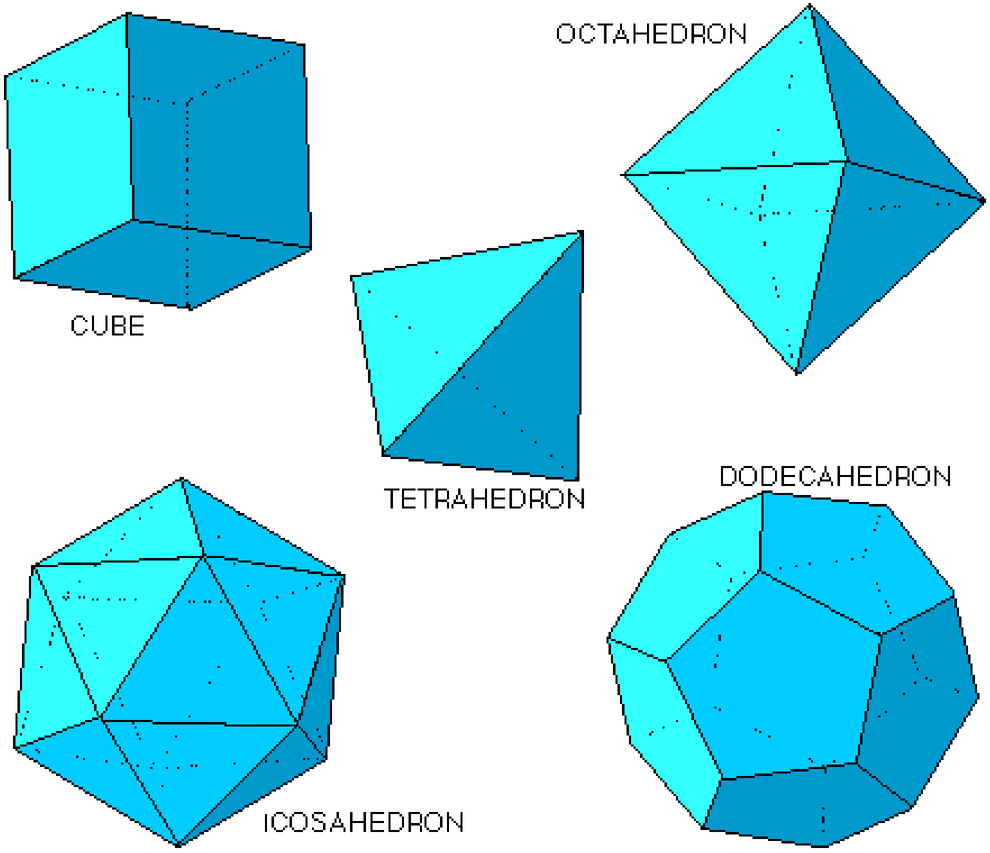




Ok, Great!

Now, what is this sonobe project good for?

Duality of Regular Polyhedra



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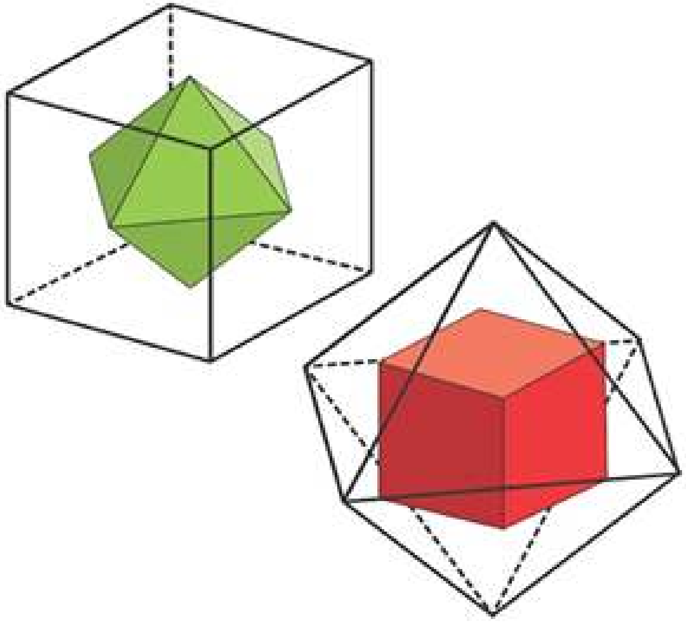
Cube with Octahedron



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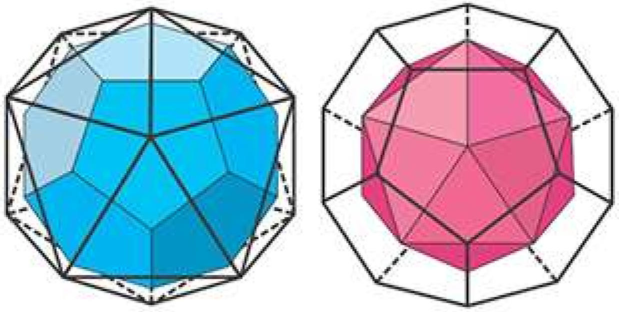
Icosahedron with Dodecahedron



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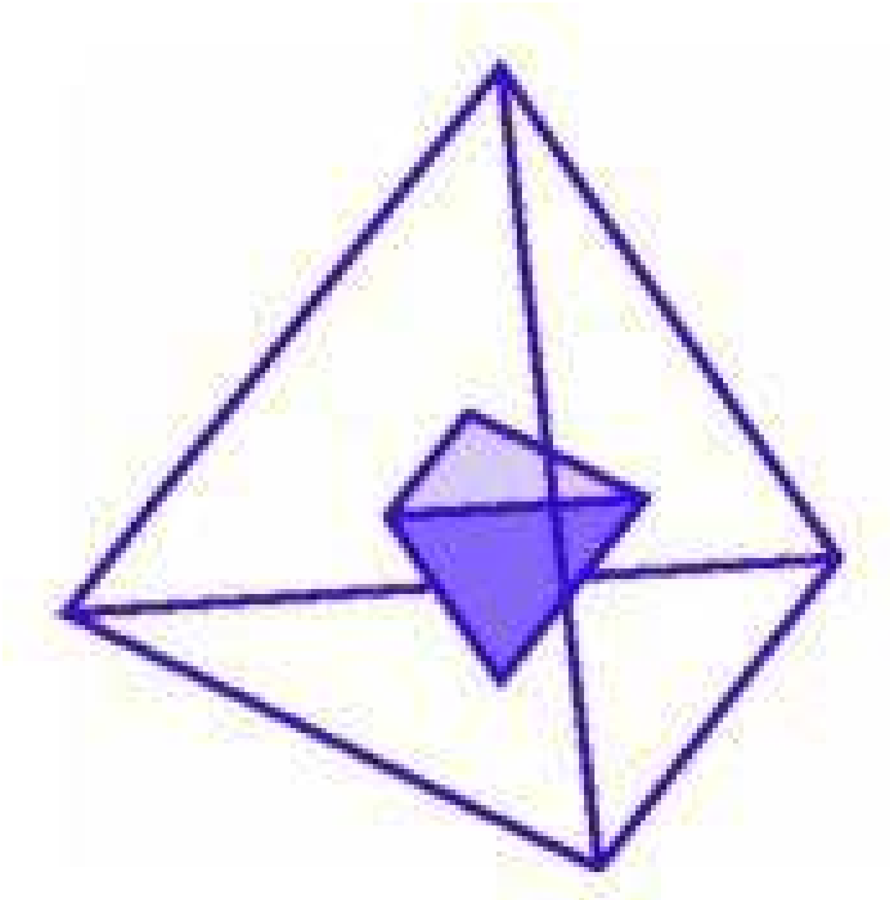
Tetrahedron is self dual



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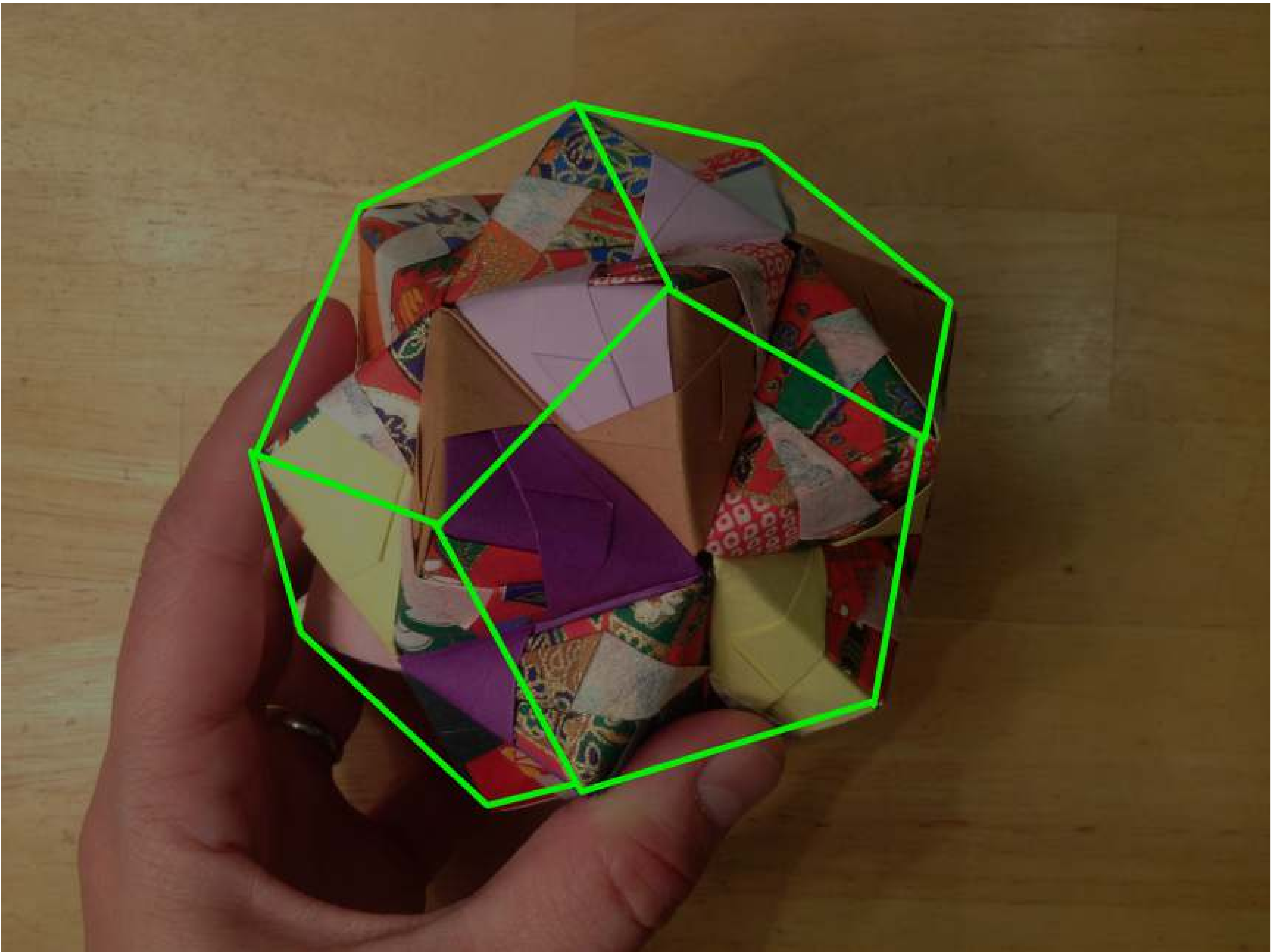

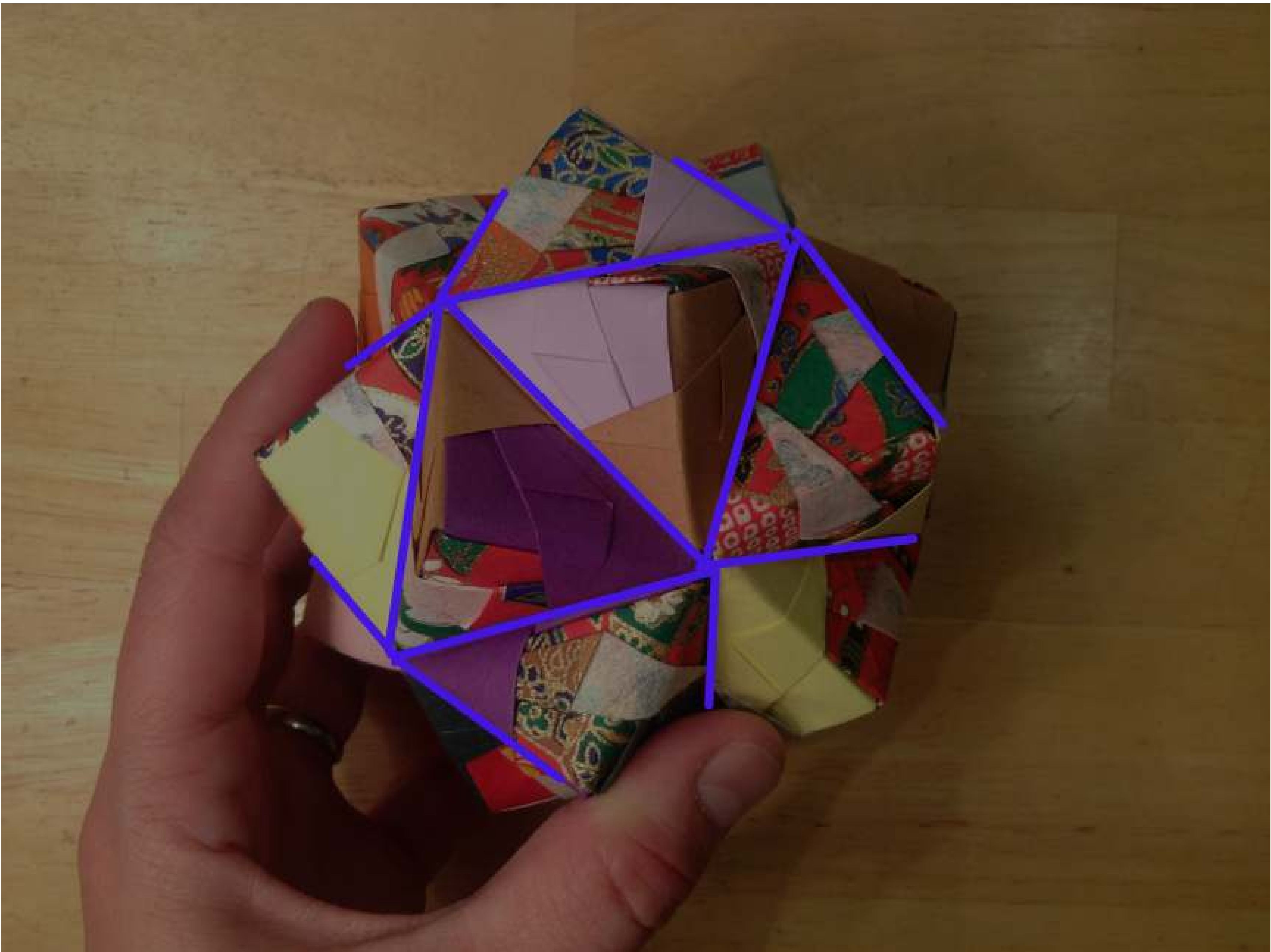

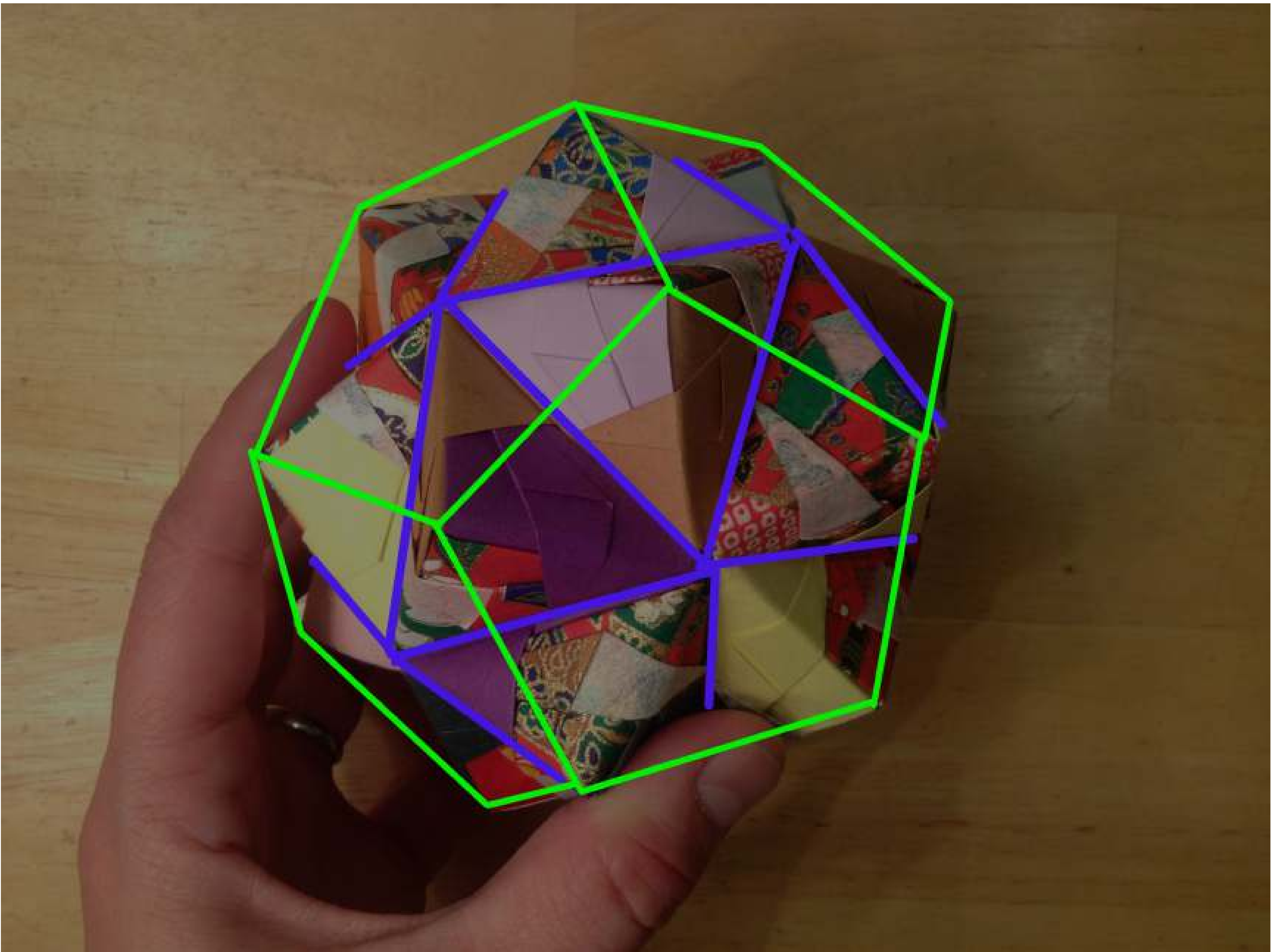

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How much paper is that!?

To build the basic object, consider the underlying icosahedron (20 sides) and count pyramids.

Each side has a pyramid attached which uses 3 units. But each unit is used in two pyramids.

$$20*3/2 = 30$$

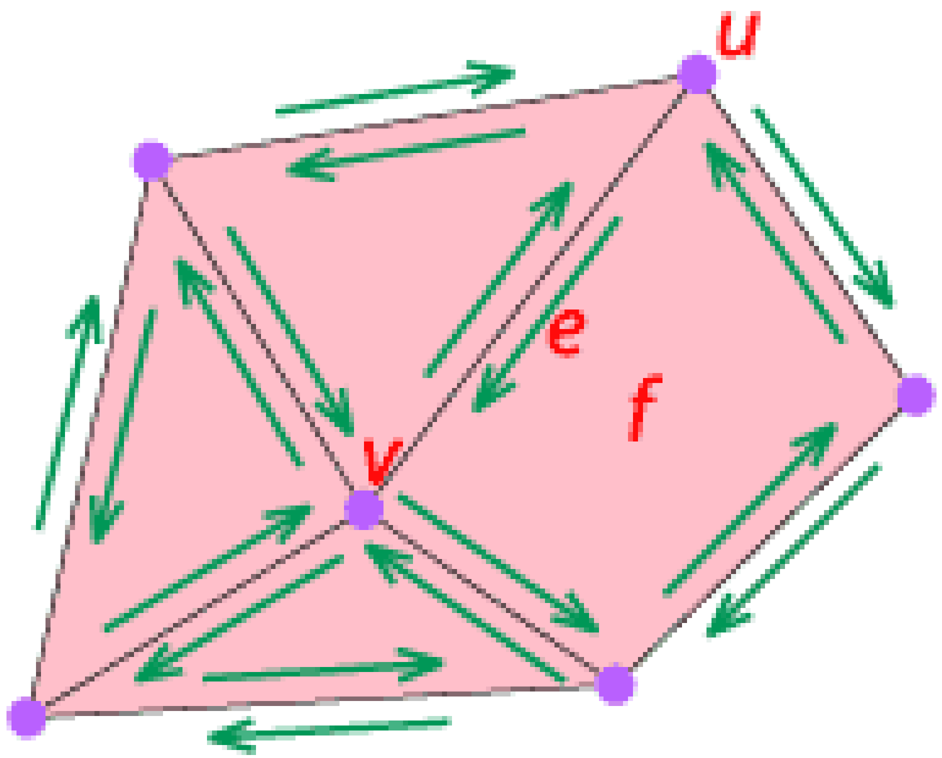
Other objects

- Tetrahedron: 4*3/2 = 6 units, but it's a cube!
- Octahedron: 8*3/2 = 12 units
- Icosahedron: 20*3/2 = 30 units
- Massive ball: (12x5)*3/2 = 90 units

All of these objects are based off of an underlying triangular mesh frame.

Euler's χ for topological classification

Euler's χ for topological classification



- 6 vertices = v
- 9 edges = e
- 4 faces = f
- $\chi = v - e + f$
- $\chi = 1$

  for single components

  with no holes

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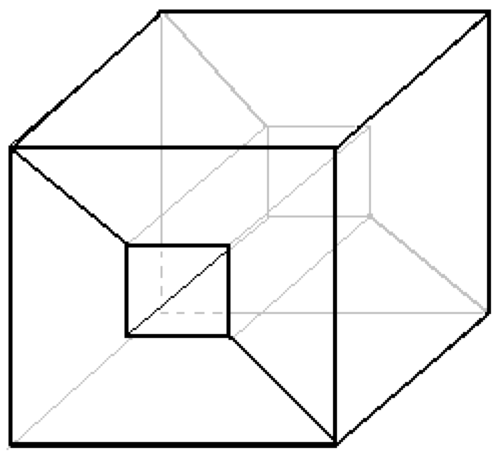
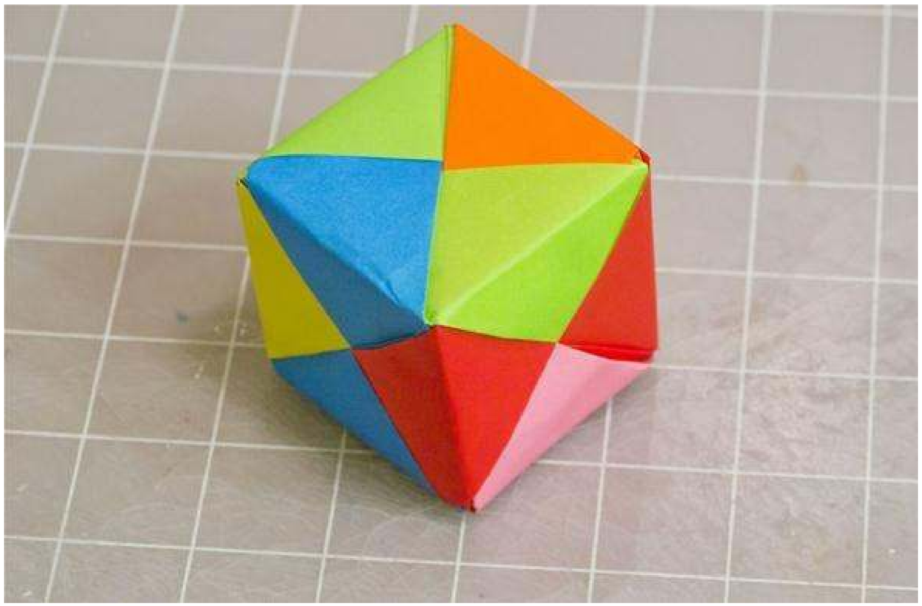
Euler's χ for topological classification



- 16 vertices = v
- 28 edges = e
- 14 faces = f
- 2 “blobs” = b
- $\chi = v - e + f - b$
- $\chi = (16+14)-(28+2) = 0$

  for “donuts”

Euler's χ for topological classification



- 8 vertices = v
- 12 edges = e
- 6 faces = f
- 1 “blob” = b
- $\chi = v - e + f - b$
- $\chi = 8 - 12 + 6 - 1 = 1$

  for “spheres”

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Euler's χ for topological classification

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- 6+8 vertices = v
- (3+3/2)*8 edges = e
- 8*3 faces = f
- 1 “blob” = b
- $\chi = v – e + f - b$
- $\chi = 14 - 36 + 24 – 1 = 1$

  for “spheres”

Euler's χ for topological classification

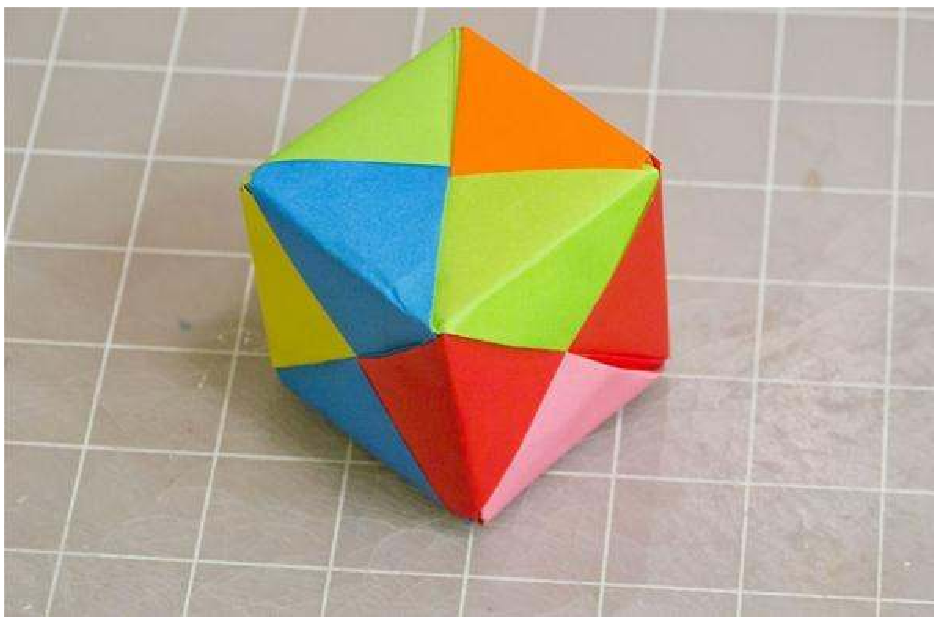
- 20 + 12 vertices = v
- (3+3/2)*20 edges = e
- 20*3 faces = f
- 1 “blob” = b
- χ = v – e + f - b
- χ = 32 - 90 + 60 – 1 = 1 for “spheres”

Angle deficit for topological classification

Angle deficit for topological classification

“And now for something
completely
different...”

Angle deficit for topological classification



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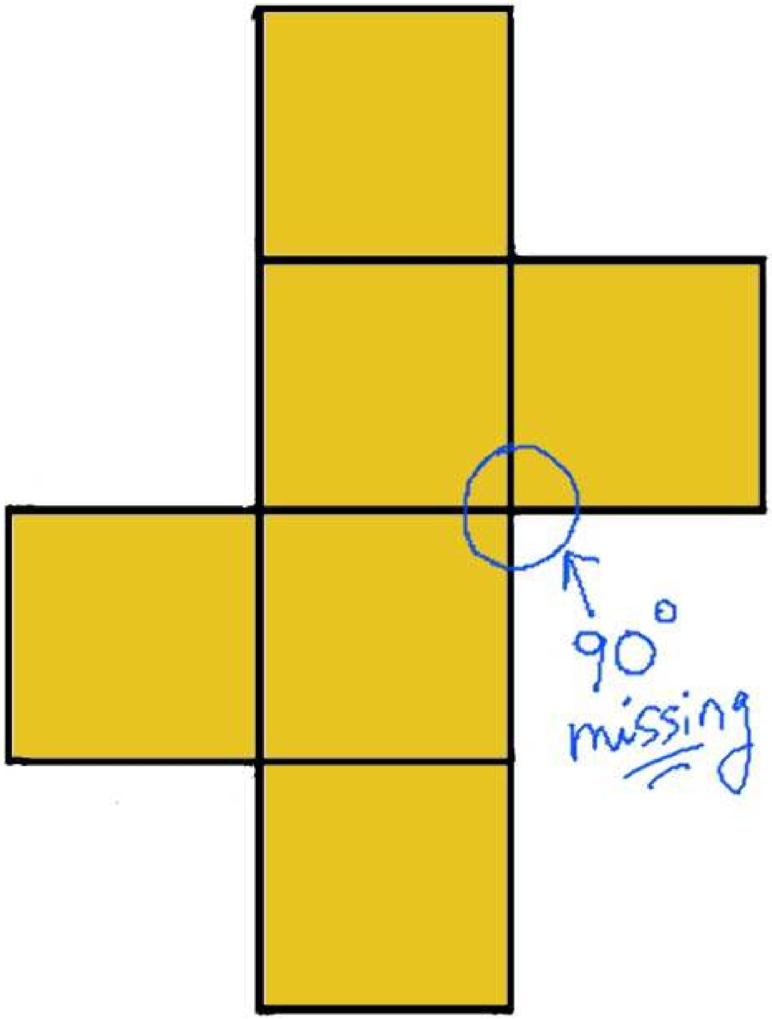
- Imagine a flattened cube
- At each vertex, 90 degrees is “missing”
- 8 * 90 = 720
- Total Angle Deficit is 720

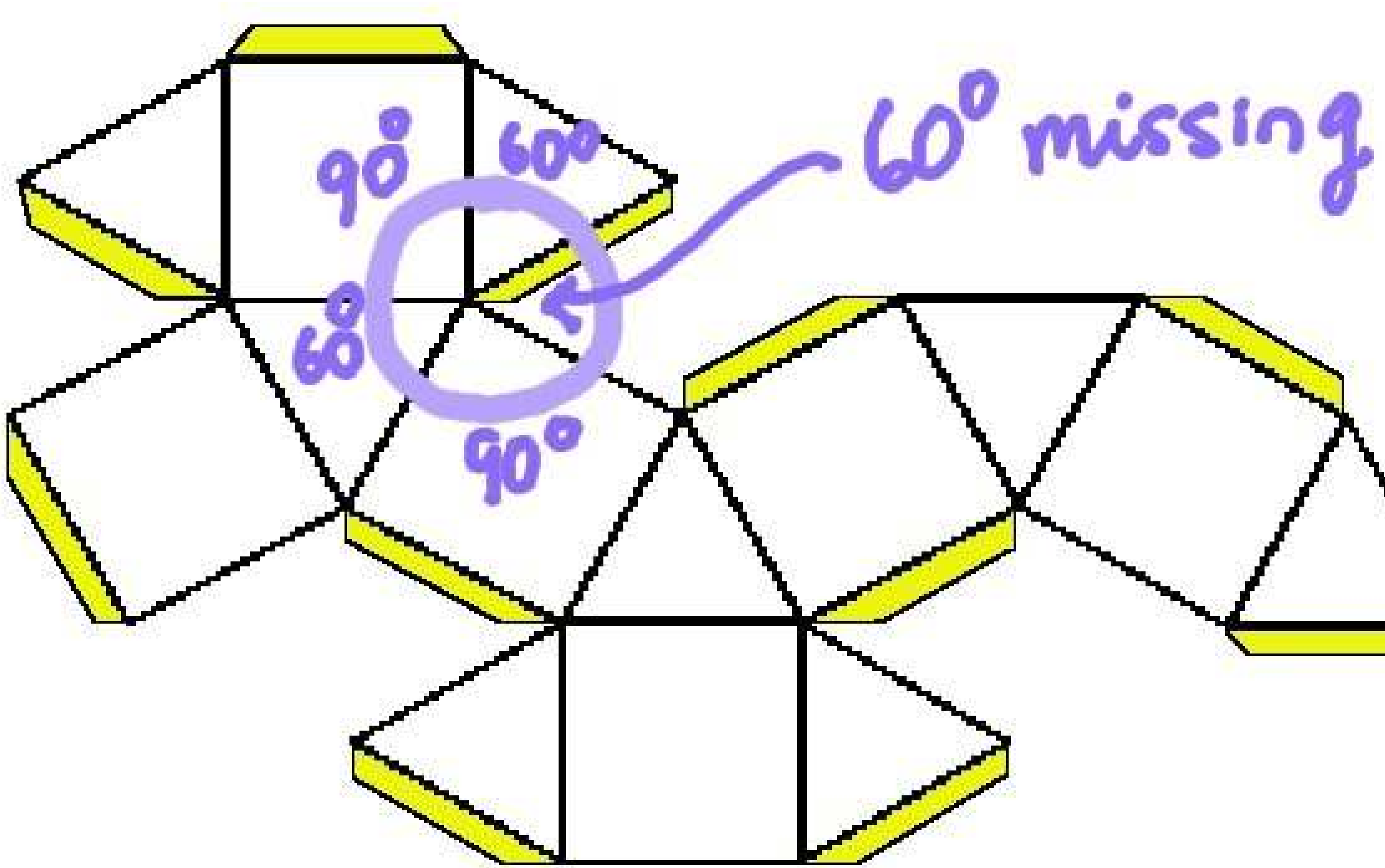
Angle deficit for topological classification



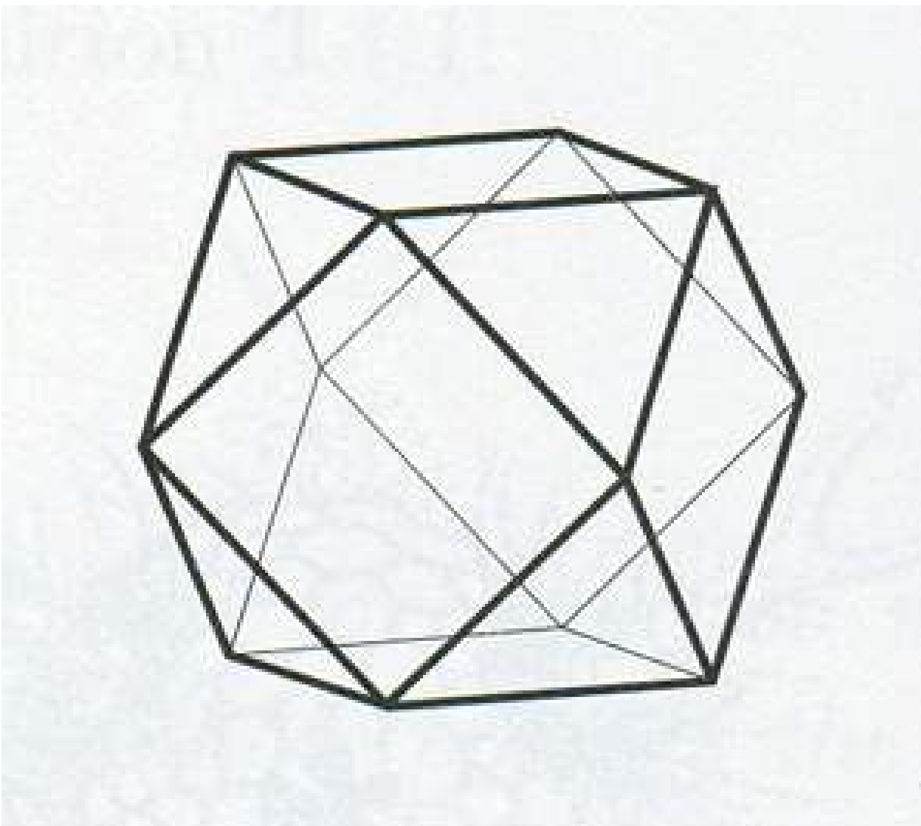
- A flattened cube!
- At each vertex, 90 degrees is “missing”
- 8 * 90 = 720
- Total Angle Deficit is

  720

Cuboctahedron Template



Cuboctahedron in 3D



- 12 vertices
- 60 degree deficit per
- 12*60=720
- Total Angle Deficit is 720

Sonobe!

- 20 vertices with 90 degree deficit (elliptic curvature!)
- 12 vertices with -90 degree deficit (hyperbolic curvature)
- 20*90-12*90=720

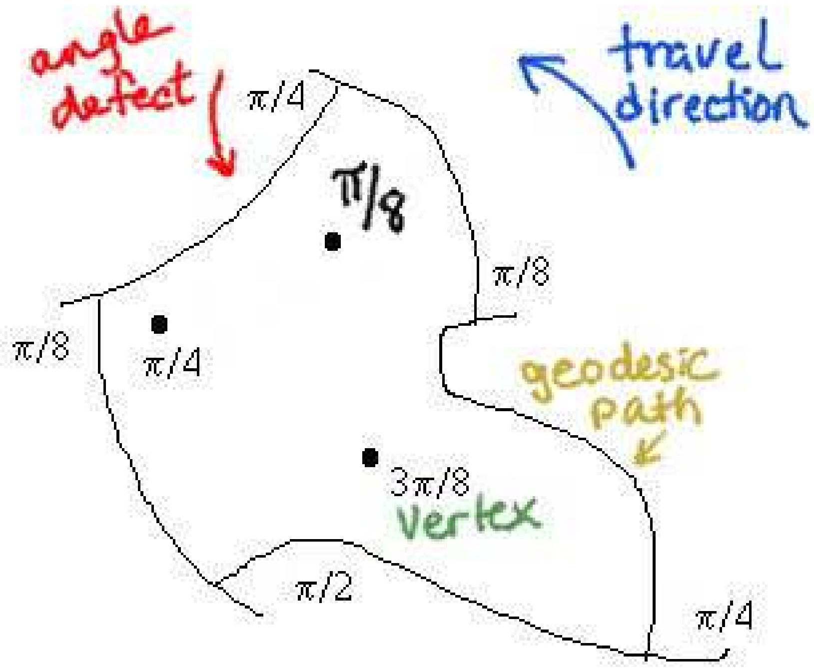
But....

$$\text{Total Angle Deficit} = 4\pi * \chi$$

Gauss-Bonnet Theorem

Gauss-Bonnet Theorem

If we trace out a closed curve on a surface, the total enclosed Gauss curvature (total angle deficit) is 2π minus the total angle defect (angles of deflection) around the curve.



Gauss-Bonnet simple example

- Total angle defect is zero. (This paper ribbon is a geodesic.)
- Each half of the sphere must have the same enclosed Gauss curvature.
- Total must be 4π
- Each half contains

  2π

Gauss-Bonnet simple example

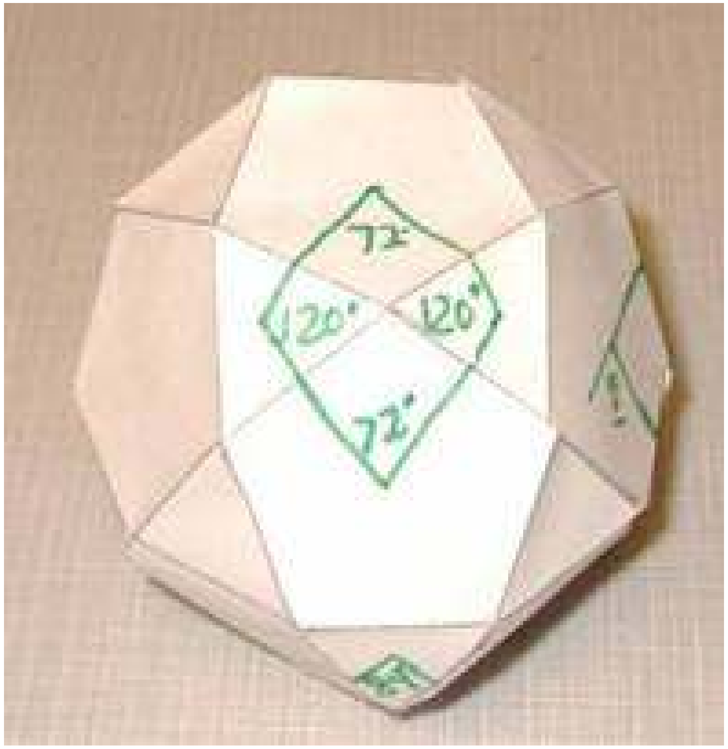
- Total angle defect is zero. (This paper ribbon is a geodesic.)
- Each half of the can must have the same enclosed Gauss curvature.
- Total must be 4π
- Each half contains

  2π

Gauss-Bonnet simple example

- Total angle defect is zero.
- Each half of the cube must have the same enclosed Gauss curvature.
- Each half contains 2π
- Each vertex has $\pi/2$

Gauss-Bonnet example



- Total angle defect is:

  2(180-72)+2(180-120)

  336
- 360-336=24 degrees
- Angle deficit:

  360-(60+60+108+108)

  24 degrees

Gauss-Bonnet example

- Total angle defect is:

  3(180-84)

  288
- 360-288=72 degrees
- Angle deficit:

  3(360-60-60-108-108)

  720 degrees

Gauss-Bonnet example

- Total angle defect is:

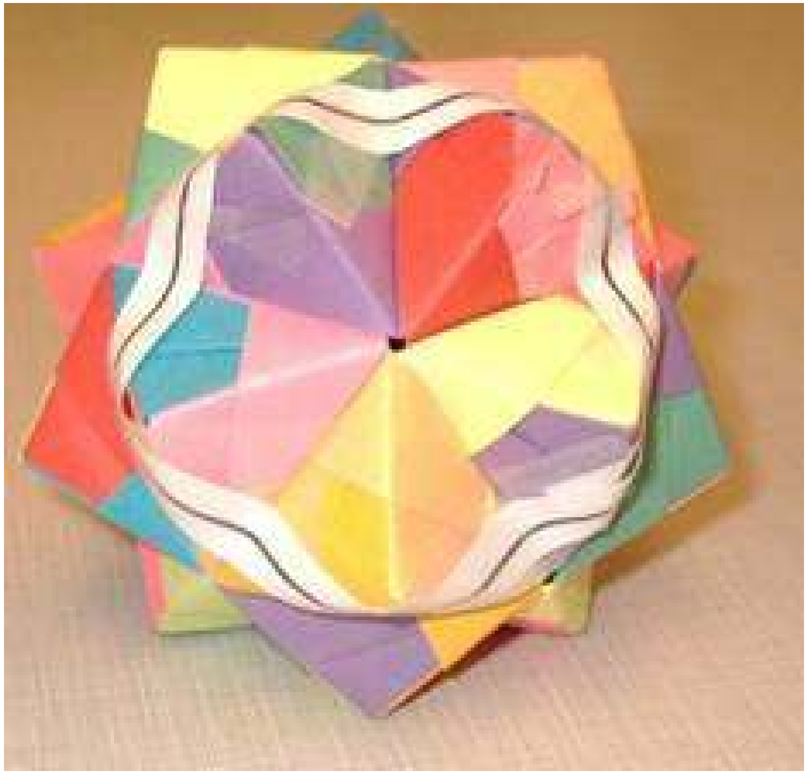
  5(180-120)

  300
- 360-300 = 60 degrees
- Angle deficit:

  5(360-108-120-120)

  60 degrees

Gauss-Bonnet example



- Total angle defect is zero (geodesic!)
- 360 degrees is promised
- Angle deficit:

  5(90)+(-90)

  360 degrees

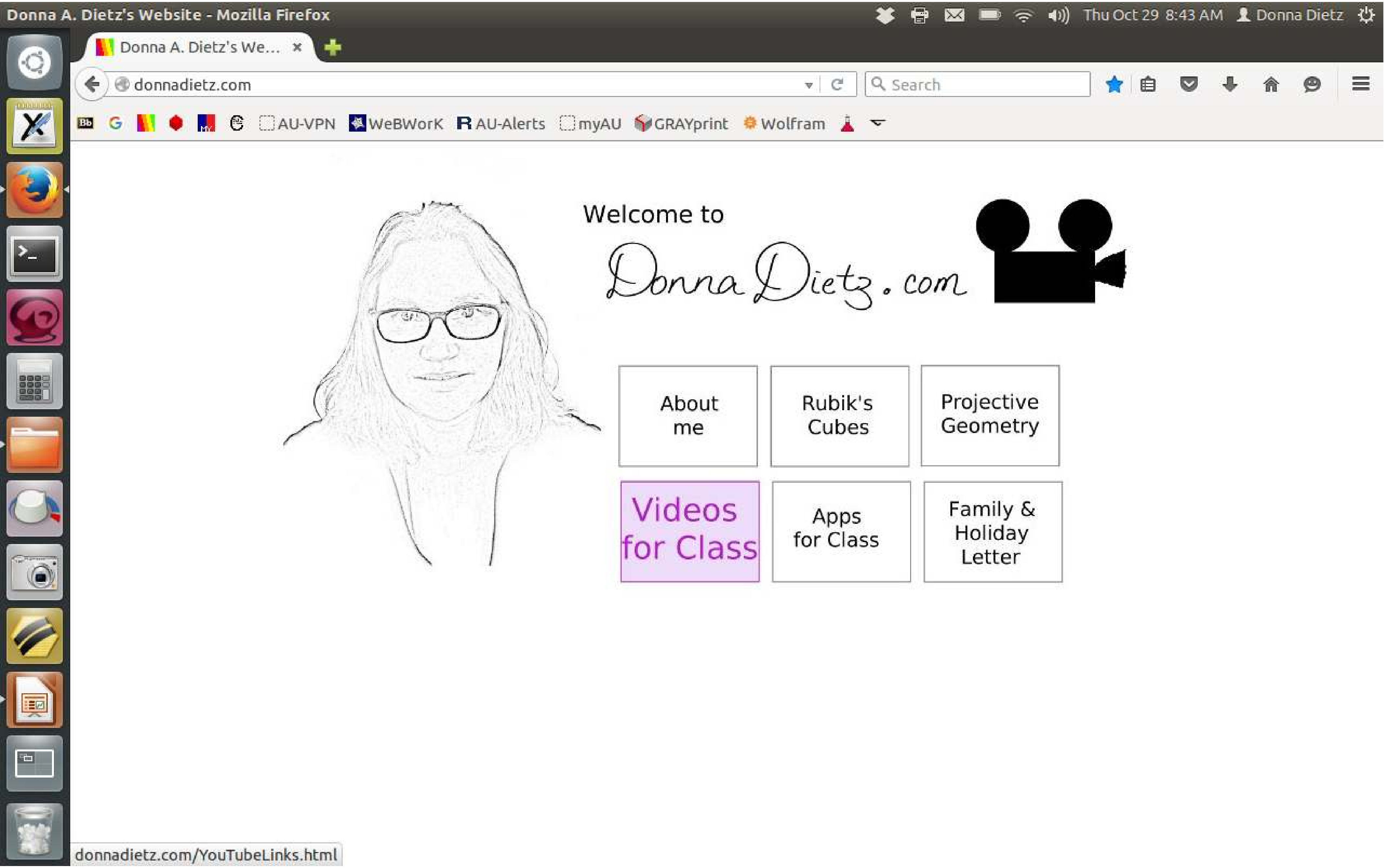
Bonus Puzzle

Use 3 colors, 10 pieces of each color.

Try to make sure you get exactly
one of each color
on each pyramid.

(Solve using a graph first.)

Bonus Puzzle Solution



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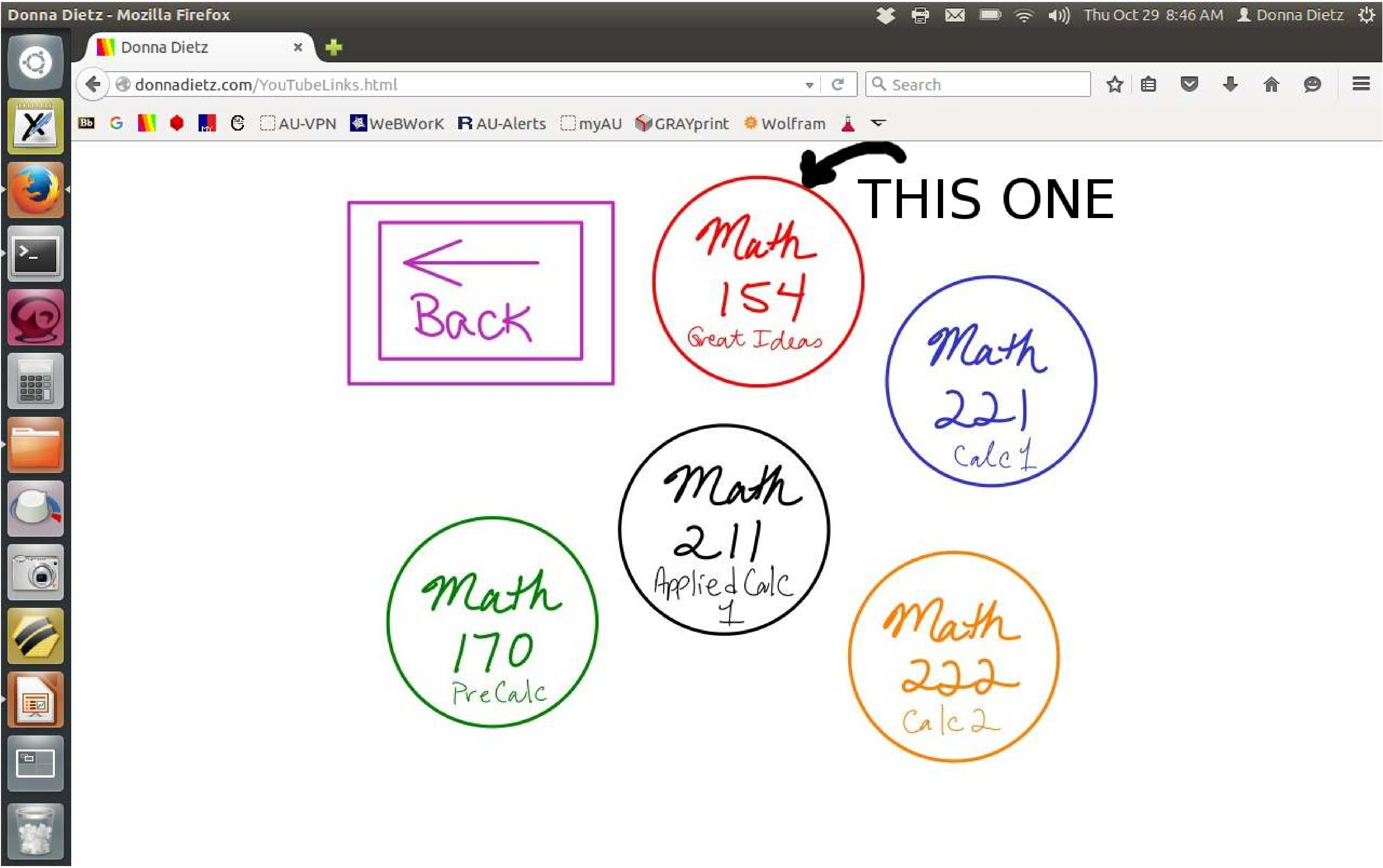
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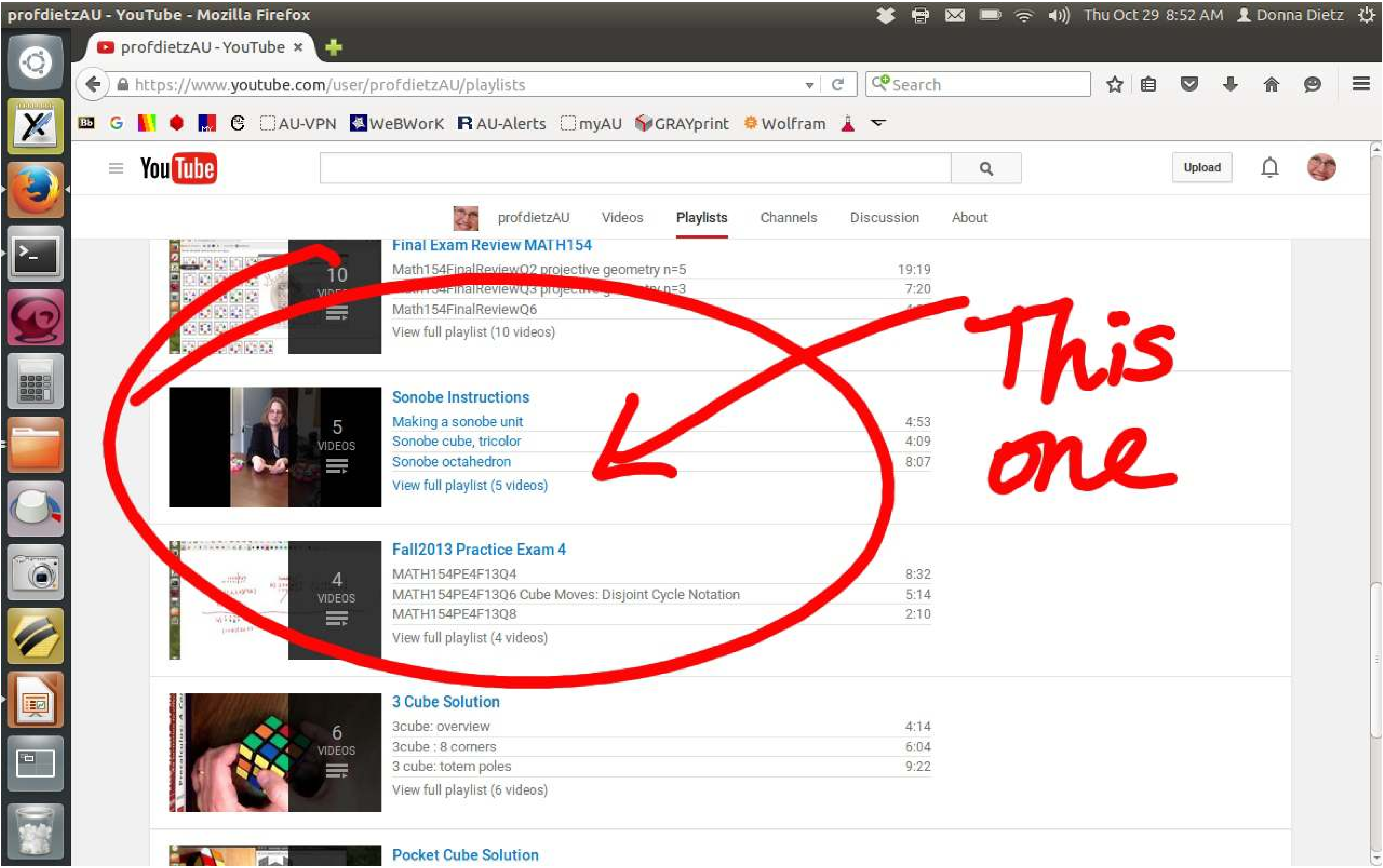
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