Twisty Puzzles
for Liberal Arts Math Courses

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The general public tends presume that anyone who can solve Rubik's Cubes (and similar puzzles) is very smart. Teaching Liberal Arts students how to solve these puzzles is a great way to improve their mathematical self-esteem while teaching them about group theory, modular arithmetic, algorithmic design, 3D geometric visualization and much more. The key realization that makes this possible is that with just 2 or 3 simplistic routines, the pocket cube (or other puzzle) can be solved. Most published solutions focus on the overall speed of solution rather than reducing human memorization. By focusing on reducing the number and complexity of the routines, solutions are within reach of all undergrads and can be mastered after just a few class periods. (Students who are good at mathematics tend to learn the pocket cube in well under an hour.)

Grab One:

Pocket Cube Solution:
http://www.donnadietz.com
1) Solve top half of puzzle
2) Position bottom corners
(UR)^7 U

U=Up face Clockw., R=Right face Clockw.
Swaps FDL with BLD. Reposition cube.

3) Rotate bottom corners
(RC)^{12}

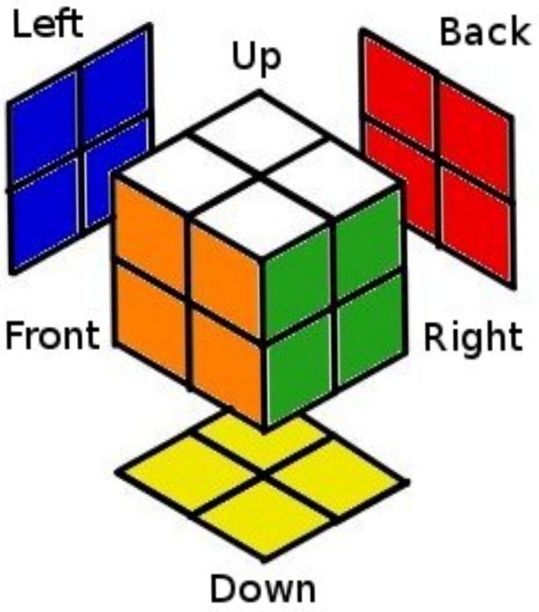
C=Turn puzzle Clockw. looking down upon it.
Rotates FDL, BLD, RBD Clockw.

a) If 0, 2 rotated correctly:
Find corner which needs to rotate
Clockw. Place in FRD position.
b) If 1 rotated correctly: Place in FRD.

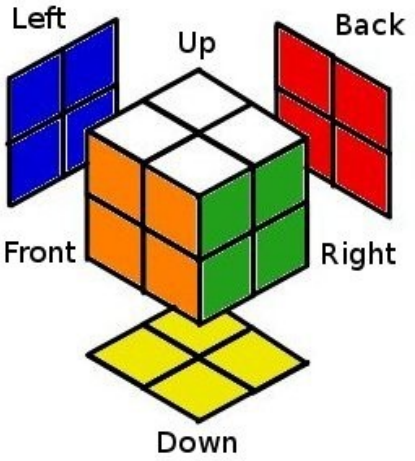
Objectives of Talk:

- Encourage you to pick up these toys again for the first time*... for your own enjoyment!
- Give you some tips for classroom use

* Kellogg's Corn Flakes commercial (1990)



The pocket cube



For the sake of time, I will only talk about this one puzzle, which seems to be a very good bet in the classroom!

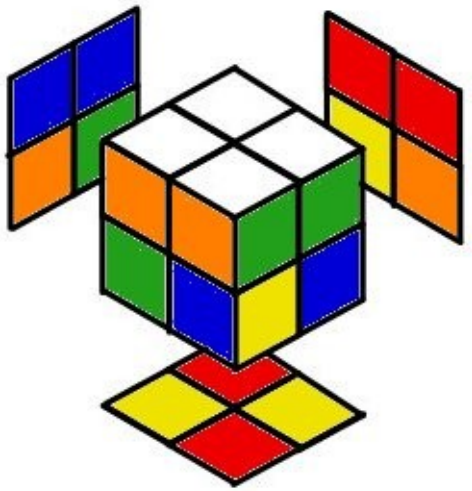
But, the ideas generalize!

And I thought I was over it by now!

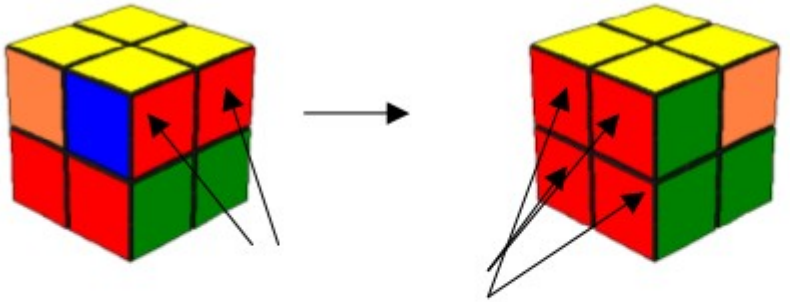
- I was a child of the 80's and into the cube.
- Like so many of us, I mastered the cube, just barely, and quickly forgot all the routines.
- I occasionally have gone through the cube phase again, but never with permanent mastery.
- I acquired a Master's student who wanted to work on twisty puzzles. (Porfirio Velasquez)
- So, I started looking at them with new eyes...

Why is it hard to solve?

- We figure out how to solve the top layer intuitively, and that intuition tends to stick with us for life.



But then...

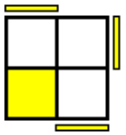


Once your cube is aligned like in my example, perform the next alg with the big square of 4 as the back face (in my example, it's red). If you don't have any matching pairs, just perform this next alg with any face (except white or yellow) as the back face.

Corner permutation algorithm: x R' U R' D2 R U' R' D2 R2 x'

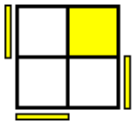
Or maybe...

Sune



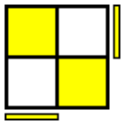
- R U R' U R U2' R'
- **O:** R D' R2 F2 D' F
- **O:** R D' R2 (y') R2 D' R
- **O:** L' D L2 (y) L2 D L'
- **G:** R2 U' R U' R

Anti-Sune



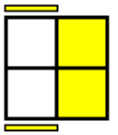
- R U2 R' U' R U' R'
- **O:** R D' R2 F2 D' F
- **O:** R D' R2 (y') R2 D' R
- **G:** R2 U R' U R

Bowtie



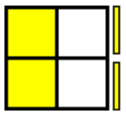
- F R U' R' U' R U R' F'
- F R' F' R U R U' R'
- R' F R B' R' F' R B
- L' U R D' R' U' R U
- **G:** R U2 R' F' U' F

T



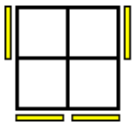
- R U R' U' R' F R F'
- R U R' U' L' U R U'
- **G:** F R U2 R' F

Headlights



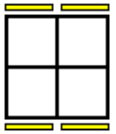
- (U2) F R U R' U' F'
- F U R U' R' F'
- R' F' U' F U R
- **G:** R U2 R U2 R

Pi



- U R U2 R2 U' R2 U' R2 U2 R
- y F R U R' U' R U R' U' F'
- **O:** R' F R2 U' R2' F R
- **O:** R U R' L' U' L R U R'
- **G:** R2 U R2 F' U2 F

H



- R2 U2' R' U2 R2
- **G:** R2 U2 R

Or...

Solution 1:

Phase 1: Solve the top layer.

a. Decide which colour you want to place on the top face. Then hold the cube so that at least one piece already shows that colour on the top face. The remaining pieces on the top face will be placed relative to that piece.
b. Find a piece in the D layer that belongs in the U layer. Rotate D so that the piece is below its destination and rotate the whole cube to get it to the front right. If all the U layer pieces are already in the top layer though some are wrongly placed then choose any D piece to displace any wrong piece from the U layer.
c. Depending on its orientation, do one of the following:
   1. To move FRD->URF do FDF'
   2. To move RDF->URF do R'D'R
   3. To move DFR->URF do FD'F'R'D2R
d. Repeat b-c until the top layer is done.

Phase 2: Place the bottom layer pieces, ignoring orientation.

a. Rotate the D layer to get at least two pieces in the correct position, though they may be twisted.
b. If necessary swap two pieces by using the following:
   1. To swap DFR, DBL do F'R'D'RDFD'
   2. To swap DFR, DFL do FDF'D'R'D'R

Phase 3: Twist the bottom layer pieces correctly.

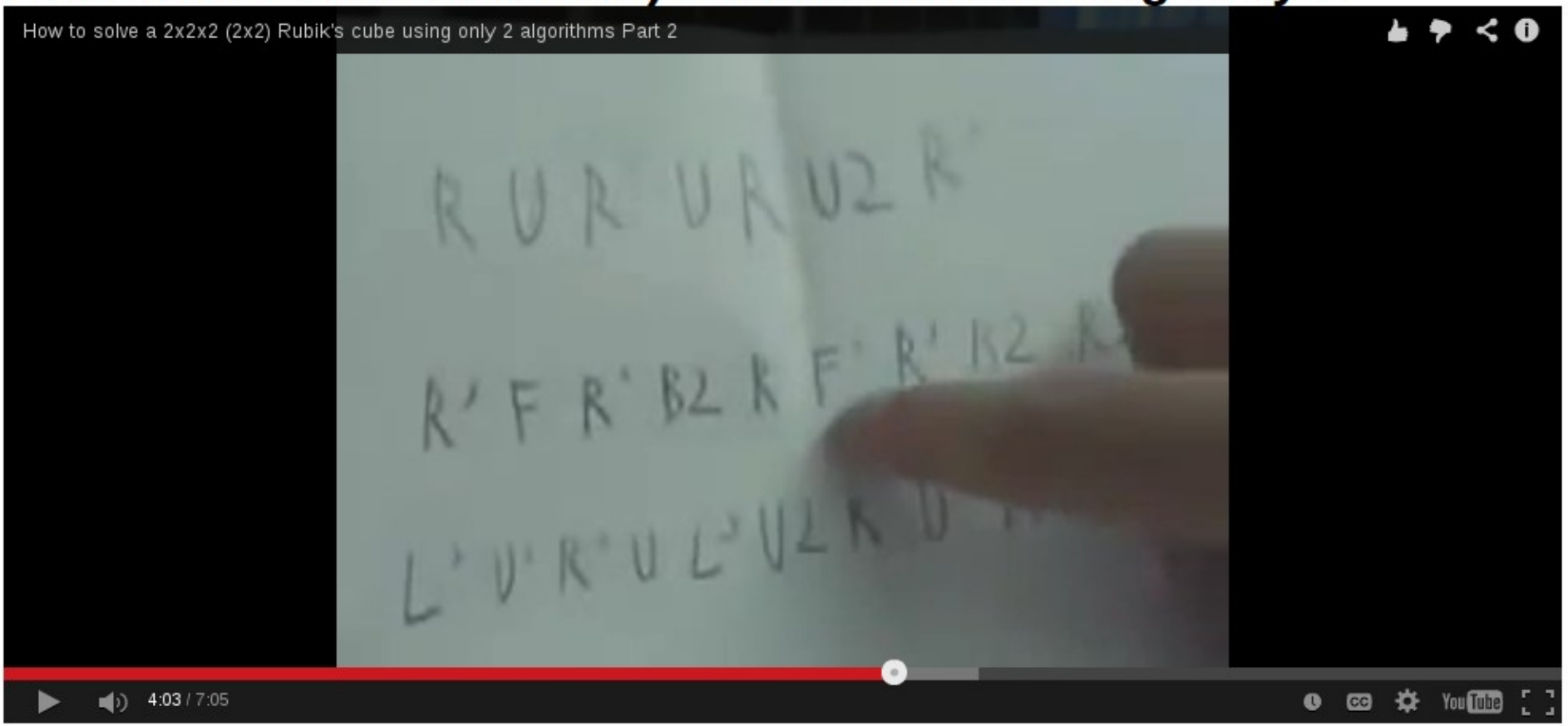
a. Do one of the following sequences to solve the remaining pieces. Clockwise twists are denoted by a +, anti-clockwise ones by a -.
   1. To twist DFL-, DBL+ do R'D'R F' DR'DR D2F2
   2. To twist DFL+, DBL- do F2D2 R'D'RD' F R'DR
   3. To twist DFL-, DBR+ do R2D'R D2R'D2R D'R2D
   4. To twist DFR-, DRB-, DBL- do R'D'RD' R'D2RD2
   5. To twist DFR+, DRB+, DBL+ do D2R'D2R DR'DR
   6. To twist DFR-, DRB+, DBL-, DLF+ do R2D2 R D2R2 D
   7. To twist DFR+, DRB-, DBL-, DLF+ do RDF R2D2F2 DF'DR2

This solution takes at most 24+8+10=42 moves.

(Jaap's Page: http://www.jaapsch.net/puzzles)

I searched for “easy” solutions:

How To Solve A 2x2x2 2x2) Rubik S Cube Using Only 2



How To Solve A 2x2x2 2x2) Rubik S Cube Using Only 2 ...
Source : http://www.youtube.com/watch?v=toLvJY3wF8A

Another “easy” solution:

How To Solve A 2x2x2 Rubik S Cube Easily

How To Solve A 2x2x2 Rubik S Cube Easily ...
Source : http://www.youtube.com/watch?v=MONNEaPTAsM

Dietz's Routines

Swapping: $(UR)^{7}U$

Rotating: $(RC)^{12}$

Cubing Obstacles

- You need to be able to maintain orientation of the object in 3D.

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- You need to be able to maintain orientation of the object in 3D.
- You need to read the routines with your eyes while manipulating the object with your hands.
- If you mess up, you need to start over from the beginning.
- Even two or three routines will be forgotten in a week.
- There is a basic lack of intuition after layer one.

What options do we have
for solving
the 2x2x2?

Ask your favorite oracle!

11 moves maximum (5 seconds)
(14 moves if using quarter-turn metric)

Quite Fast:

24 moves max. (10 seconds)

(see Jaap's Page)

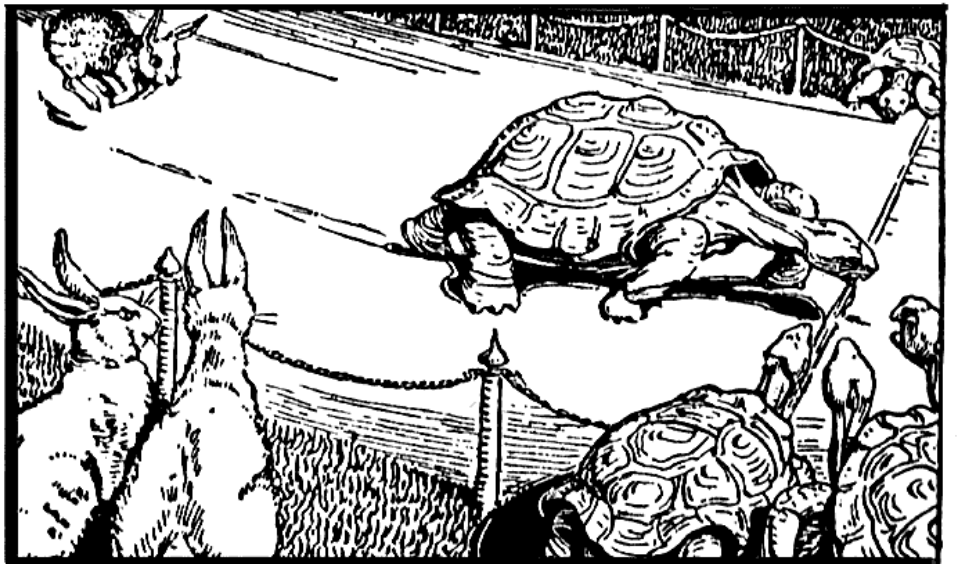
Moderate Speed:

42 moves max. (20 seconds)

It's a trade-off:

- Fast!
- Many routines.
- Hard to recall later.
- Black Magic.

- Slow down.
- Fewer routines.
- More intuitive.



Let's slow down!

- Top layer should be intuitive. This part, most people remember years later even without practice.
- Next, position the bottom corners

  with one routine.
- Last, rotate the bottom corners

  with one routine.
- Maybe 100 moves?

  (1 or 2 minutes)

First Routine:

$(UR)^7\ U$

Used only in stage two.
Usually only used once.
Swaps FLD with BDL
“the 15 move”

Second Routine:

$(RBLF)^3$

Second Routine:

$$(RBLF)^3$$

Second Routine:

$(RC)^{12}$

Used only in stage three.
Used up to four times.
Rotates three lower corners (not FRD) clockwise.
C=turn whole cube clockwise (looking at U).
“the 12 move”

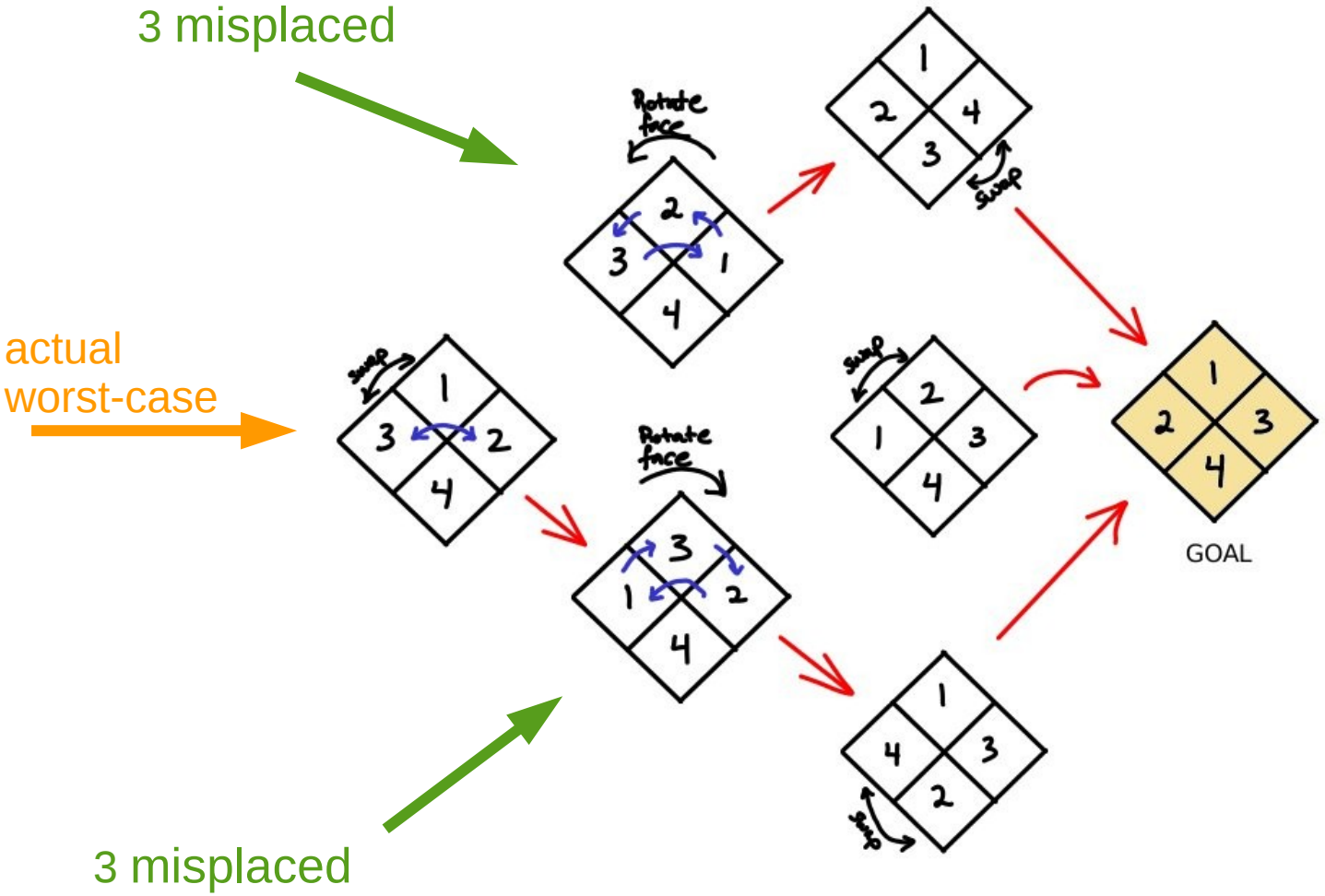
Removes these barriers

- You can look at your hands while performing the routines. In fact this is useful to do!
- It's ok to lose track of where you are in the routine.
- If you derail, you often stay at the same stage.
- All motions are clockwise (or can easily be adjusted for left-handed students so they are all counterclockwise).
- It is much more intuitive than most solutions.
- It's very hard to forget these routines.

These barriers remain

- Students may still lose orientation (but this is less common since they can watch their hands)
- It still takes some effort for many people.
  - About an hour or so for people who already can do the top layer
  - Plan about 5 - 6 hours of classroom time for Liberal Arts students with 25 to 28 students.

Overview of Theory: Placing bottom pieces



Theorem for twisty cubes:

The sum of the twists on the eight corners
=
0 mod 3

Theorem for twisty cubes:

The sum of the twists = 0 mod 3

* if we have three twisted corners,
all are in the same direction

* there cannot be just one twisted corner

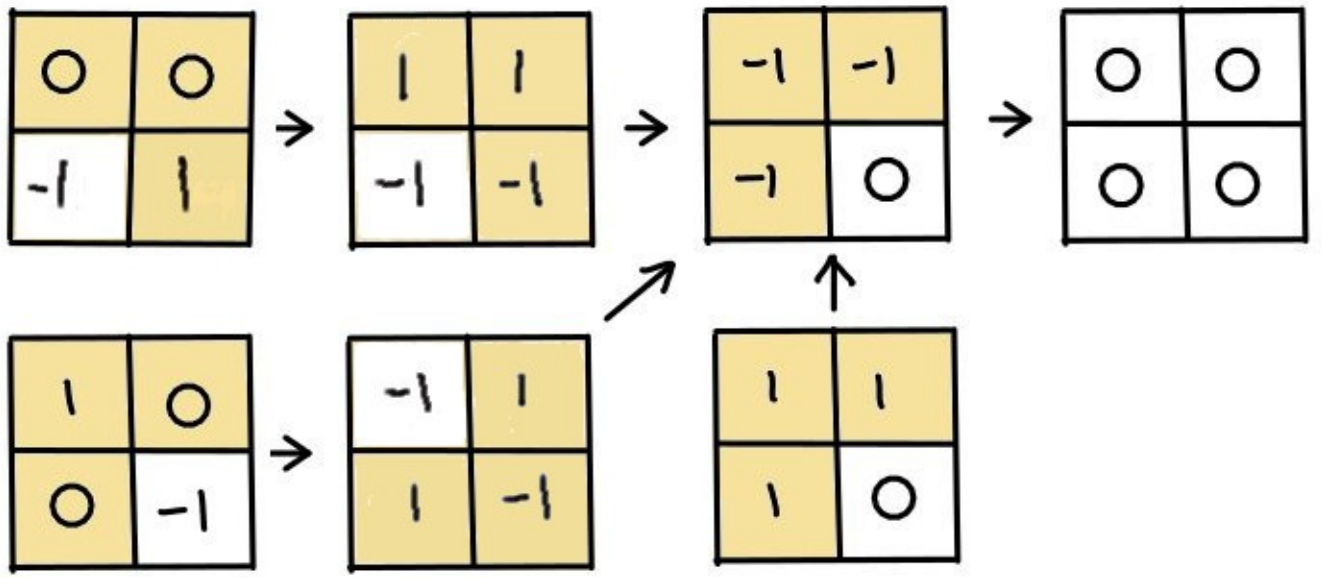
*if an even number are twisted,
half are clockwise, half are counterclockwise

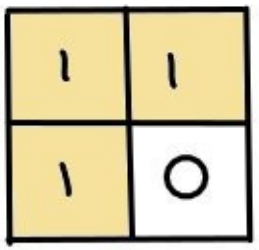
And...

$(RC)^{12}$

is a generator!

Overview of Theory: Rotating bottom pieces





One clockwise rotation is repeatedly added to three of four bottom corners until bottom corners are all rotated correctly.

Student Reactions

- Last Fall: Students insisted we learn the 3x3x3 cube after learning the 2x2x2.
  - This was fun for many, but it took longer than I had hoped. All learned the 2x2x2, but 8 of 26 didn't fully master the 3x3x3
- This Fall: Introduction to the 2x2x2 came after the Pyraminx. The students and I both felt that the experience with the Pyraminx helped them acclimate to the 2x2x2 much faster.

Your Homework:

- Buy a Shengshou 2x2x2 (or LanLan) for about $3 online.
- Use my Cheat Sheet
- Other materials and videos are on my website: http://www.donnadietz.com
- Email: dietz@american.edu
- ENJOY!!!!

Donna A. Dietz's We...

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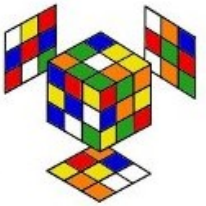


About me

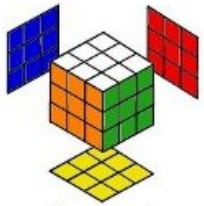
Rubik's Cubes

Projective Geometry

Videos for Class

Apps for Class

Family & Holiday Letter



donnadietz.com/Rubiks.html

http://www.donnadietz.com