Mathematical Association of America MD-DC-VA Section, October 26 & 27, 2012 Virginia Military Institute Abstracts

Invited Addresses

FRIDAY WORKSHOP

Caren Diefenderfer, Hollins University Jan Minton, Roanoke College *Creating Models to Explore Hyperbolic Geometry* 4:00 PM, Room 119, Mallory Hall

The workshop will provide some history of hyperbolic geometry and allow participants to explore two theoretical models of hyperbolic geometry (The Poincare Disk and the Open Half Plane) via Geogebra, an open source software. Bring your laptop. Participant will also create physical models of the hyperbolic plane and materials will be provided. Participants may choose whether they wish to knit, crochet or work with paper and scissors. No previous experience with knitting or crochet is necessary. We will ask those who do knit/crochet to help beginners.

BANQUET ADDRESS

Caren Diefenderfer, Hollins University Jan Minton, Roanoke College Hyperbolic Crochet Coral Reef Comes to Southwest Virginia 8:00 PM, Activities Room, Moody Hall

Minton and Diefenderfer will discuss the chronology of the Roanoke Valley Reef and describe key ideas of hyperbolic geometry. The talk will highlight the multifaceted nature of the Coral Reef project and opportunities for collaboration with other academic disciplines and the wider Roanoke Valley community.

SATURDAY INVITED ADDRESSES

David Lyons, Lebanon Valley College *Quantum Information: An Ongoing Research Program with Undergraduate Students* 9:45 AM, Activities Room, Moody Hall

This talk introduces the beautiful subject of quantum information that involves physics, mathematics, and computer science. We examine the intuition-defying topic of quantum entanglement---dubbed "spooky action at a distance" by Einstein---wherein physical systems, possibly separated by distances far enough to prevent interaction over short time spans, can nevertheless exhibit correlations and perform communication tasks not possible in classical models. We describe recent and ongoing joint student-faculty work in quantum information science conducted at Lebanon Valley College, with thoughts and observations about student research in particular.

Randy Cone, Virginia Military Institute A Forum on the Status of Mathematics Education: A Conversation in Solutions 2:05 PM, Activities Room, Moody Hall

Several recent rankings, studies, and articles have indicated that STEM preparedness in the United States, at all educational levels, is on the decline. Do such findings indicate the actual state of affairs in mathematics education? Perhaps more salient and manageable questions are: are we satisfied with the current state of preparedness in mathematics within our current population of students? If not, what are some possible courses of action that may we take to meet these students at their level? The point of this forum is to begin a conversation, at the MD-DC-VA Section level, about specific solutions some institutions are developing in the face of a perceived lack of mathematical preparedness in their incoming students. First, to give the forum some context, some general information from recent studies on post-secondary mathematics education will be presented. Second, the forum moderator will give some detailed information about specific programs of action that are underway at his home institution; programs of action designed to meet student-preparedness challenges. Finally, the forum moderator will ask the session attendees to

contribute their own solutions, encouraging them to speak about initiatives at their home institutions which are designed to meet the issues of student-preparedness.

Jeff Suzuki, Brooklyn College *Mini-Workshop: How to Win an Election Without really Trying* 2:05 PM, Board Room, Moody Hall

In 1812, a new word joined our political lexicon: the gerrymander, a bizarrely shaped district designed to give one political party an advantage in the next election. Since then, gerrymanders have been denigrated as "pathologies of democracy," and various schemes have been suggested to combat gerrymanders. But to date, identifying gerrymanders has barely gotten past the "I know it when I see it" stage. We'll take a look at the basic problem, and examine several different measures of bizarreness. The material can be incorporated in courses from remedial algebra, through probability and statistics, and on to undergraduate research.

Contributed Papers by Author (non-student)

Abdinur Ali, Norfolk State University Mushtaq Khan, Norfolk State University *Mathematical Aspects of Electromagnetic Band Gaps* 8:50 AM, Room 413, Mallory Hall

Electromagnetic band gap is an artificial structure which supports or suppresses certain frequencies in a specified region. The structure is made up of dielectric material sandwiched between two metallic structures. The top structure is made up of square metallic patches and there is gap between the patches. The center of each patch is shorted to a second metallic structure on the bottom side. The entire structure acts as electromagnetic band gap region, that frequency is not allowed to propagate. This paper explains the mathematical aspects of electromagnetic band gaps. The mathematical model for the band gap covers Maxwell's equations, characteristics of the medium, derivation of the wave vector and decay constants for surface waves on a general interface, boundary conditions, derivation of the impedances for the TE & TM waves, artificial magnetic conductors, derivation of the wave vector for TM & TE using artificial magnetic conductors, band gaps, reflection coefficients of an antenna with a flat metal ground plane and antenna with an artificial magnetic conductor ground plane, and electric field plane or magnetic field plane radiation distributions for a patch antenna with an ordinary ground plane and with an artificial magnetic conductor ground plane.

Ezra "Bud" Brown, Virginia Tech *The Symmetries of (7, 3, 1)* 11:30 AM, Room 312, Mallory Hall

The (7,3,1) block design is a collection of seven 3-element subsets ("blocks") of $V=\{1,2,3,4,5,6,7\}$. Each element of V is in exactly 3 of the blocks, and each pair of distinct elements is in exactly 1 block together. This beautiful design shows up in many different disguises throughout discrete mathematics, but this talk is about the symmetries of the design. These are permutations on the set V that preserve the blocks set-wise. We'll count the number of symmetries with the use of a magical theorem from algebra called the Orbit-Stabilizer Theorem. How many are there? Well, come and find out!

Dimplekumar Chalishajar, Virginia Military Institute *Multiple Ways of Solving Higher Order Differential Equations* 11:05 AM, Room 313, Mallory Hall

The Ordinary Differential Equation (ODE) is one of the fundamental branches of Mathematics. Most of the real world problems and some of the scientific problems are converted in terms of different notions of differential equations. I would like to present multiple methods of solving higher order ODE using infinite series and simple algebra.

David Clark, Randolph-Macon College Geometric Topology for Liberal Arts Majors 3:40 PM, Room 313, Mallory Hall

Some of the most beautiful ideas in mathematics live in the realm of geometry and topology, fields that are usually offlimits to any undergraduate except an advanced math major. How can we make the wonders of dimension, orientation, manifolds, and invariants accessible to a liberal arts major? It may be easier than you think.

James Cook, Liberty University

Where Did the "x" Come From?

9:15 AM, Room 412, Mallory Hall

The inspiration for this talk is the question familiar to many students and probably all teachers of differential equations: "where'd the x come from?". In particular, the solution of ay"+by'+cy=0 has an x appear in the double-root case. In contrast, for the Cauchy-Euler problem we find a mysterious ln(x) appear. We discuss several answers and some history of this problem.

Wayne Dymacek, Washington and Lee University

Playing at Mathematics: A Course on the Mathematics of Puzzles and Games

4:05 PM, Room 312, Mallory Hall

In this talk I will describe my attempts at teaching a course on the mathematics of puzzles and games. This is not a course on game theory and the primary emphasis is not on probability and statistics. The basic question in the course is "Can mathematics help me play this game better?"

Barb Freeman, Poquoson High School

The Wheels on the Bus

11:05 AM, Room 413, Mallory Hall

Session will explain one simple connection whereby all types of problems concerning angular and linear velocities, rpm, and pulley systems can be worked. One simple diagram will be given. Twenty minute session will explain the diagram and answer the age-old question from students: Why do we have to use radians?

Jennifer Galovich, St. John's University and the College of St. Benedict

The Thesis that Wasn't

8:50 AM, Room 412, Mallory Hall

A sequence of integers is unimodal if it increases to some point and then decreases. I will discuss a family of sequences that I am morally certain is unimodal, what those sequences are counting, and various approaches to proof of unimodality, none of which have succeeded (yet!).

Maila Brucal Hallare, Thomas Nelson Community College Undergraduate Explorations: Lattice Differential Equations 3:15 PM, Room 312, Mallory Hall

What are lattice differential equations or LDEs? Students in their final years in college will hear about the so-called ODEs and PDEs, which are ordinary and partial differential equations, respectively. In this talk, I will present an introduction to LDEs, which can be viewed as differential equations that are "between" ODEs and PDEs. Like ODEs, the component equations involve derivatives with respect to a single variable; like PDEs, these systems are infinite-dimensional in nature. We will look at a canonical example of an LDE, its real-life applications, and existing basic results.

Gregory Hartman, Virginia Military Institute

APEX Calculus: A Progress Report

11:30 AM, Room 412, Mallory Hall

VMI recently awarded the presenter with course releases so he could collaboratively write an open source Calculus text which is currently in use in VMI's Calculus I course. Work on the project is ongoing; currently, material for Calculus II is being prepared. This talk gives an overview of the project. Ample opportunity for suggestions and discussion will be given.

Heidi Hulsizer, Hampden-Sydney College Mathematics in Call of Duty: Black Ops? 4:05 PM, Room 413, Mallory Hall

Call of Duty: Black Ops is a popular video game with no apparent educational value. However, one can use mathematics to solve an interesting problem that appears in the game. The problem involves turning dials to reach a solution, but turning one dial also turns others. Two perspectives on the solution will be presented; one involves solving a system of equations modulo ten and the other involves converting a directed graph with four vertices into a matrix equation. This example provides an affirmative answer to the question, will I ever use math in "real" life?

Inhan Izmirli, George Mason University *An Elementary Proof of Mean Inequalities* 8:50 AM, Room 312, Mallory Hall

In this paper we will extend the well-known chain of inequalities involving the Pythagorean means, namely the harmonic, geometric, and arithmetic means to the more refined chain of inequalities by including the logarithmic and identric means using nothing more than basic calculus. Of course, these results are all well-known and several proofs of these and their generalizations have been given. See Hardy, Littlewood, and Polya (1964); Carlson (1965); Carlson and Tobey (1968); and Beckenbach and Bellman (1971); Alzer (1985a, 1985b) for more information. Our goal here is to present the proofs as corollaries of the same simple theorem.

Lydia Kennedy

An Algebra with Characteristic Dependent Associativity

11:30 AM, Room 313, Mallory Hall

Let K be a field. We will construct an algebra V over K with the property that the associativity of multiplication is dependent on the characteristic of K. We explore other properties of the algebra including the annihilator ideals, an associated Lie algebra and the ideal lattice.

Chris Lee, Roanoke College

Technology in a Mathematics Classroom: Tablet PC's, Moodle Integration, and Mathematica 3:40 PM, Room 413, Mallory Hall

In the Mathematics department at Roanoke College we have always involved technology extensively in our classrooms. This paper session will be a case study on the successful implementation of a variety of technologies in a mathematics course. While all have been successful, important pros and cons of adoption of technology will be presented. Components of my paper session will include the following. Tablets PCs – their incredible usefulness in a mathematics course. I have not written on a chalkboard in over 5 years and only use a tablet PC in class. All of my notes are written live in electronic form and posted for students, and much of my grading is done electronically on work students upload through Moodle. In this respect, I will highlight how the incorporation of technology such as a tablet PC can greatly enhance a mathematics course, and integrate seamlessly with a LMS such as Moodle. Moodle Integration – how to successfully integrate and leverage a LMS in mathematics courses. At Roanoke our department has been working together to very successfully incorporate Moodle in a wide range of courses, from Calculus to Discrete Mathematics to Differential Equations. Drawing from this experience I will present concrete examples of LMS use in mathematics courses, specifically addressing the following topics: 1) Organizing a course by content or timeframe, 2) Calculus Labs – distribution and paperless grading, 3) Online quizzes as a replacement for clickers, and 4) Digital class notes. Mathematica – changing the examples and problems we can work in class. At Roanoke College, Mathematica has been our software of choice for

years. With a tablet PC running live at all times, I am able to effortlessly include Mathematica's powerful calculations and graphics in to daily classroom activity. This has greatly changed the complexity of problems that can be tackled in courses ranging from calculus to applied differential equations.

Larry Lehman, University of Mary Washington Suborders of Quadratic Polynomials Modulo Primes 4:05 PM, Room 412, Mallory Hall

If f(x) is a monic quadratic polynomial with integer coefficients and p is a prime number not dividing the constant coefficient of f(x), we define the suborder of f(x) modulo p to be the smallest positive integer m for which f(x) divides some polynomial x^m -d in the ring of polynomials over a field F with p elements. We show that there is an element of F that we can associate to f(x) that determines its suborder modulo p, thus defining a suborder function on F. We establish several properties of this function, particularly in connection to the periodic nature of a particular linear recurrence relation defined over F.

Caroline Grant Melles, United States Naval Academy

What is a Berkovich Curve?

3:40 PM, Room 412, Mallory Hall

Berkovich curves over non-Archimedean fields are beautiful geometric objects with connections to algebraic geometry, tropical geometry, and number theory. To each smooth projective algebraic curve X over a non-Archimedean field K, we associate a Berkovich analytification X_{an} . We visualize K as a totally disconnected set, such as the *p*-adic numbers, and imagine connecting the K-points of X by growing edges from each point to connect to a "skeleton" in X_{an} . When X has genus at least 1, there is a unique "minimal skeleton" in X_{an} , with the structure of a finite metric graph. The talk will conclude with some results of joint work with Amy Ksir on automorphisms of Berkovich curves and their skeletons.

Edwin O'Shea, James Madison University Hurrah for the Daily Quiz!

11:30 AM, Room 413, Mallory Hall

I used to give quizzes on a weekly basis to my freshman calculus sections. You know the routine: panic on day before the quiz, deer in headlights day of, kick back and chill (in the parlance of our times) on the other days. I wanted to give my students motivation for doing mathematics on a daily basis rather than cramming facts on a weekly basis. I'd like too to get decent daily feedback on what the students are stuck on. At a minimum, I want and need them to keep in step with the daily content. Instituting a daily quiz has partially solved these and other problems. The grading is hellish though, right? I'll give an overview of this regimen and some practical thoughts on how to implement it, including how to grade efficiently.

Marcus Pendergrass, Hampden-Sydney College Two Musical Orderings

3:40 PM, Room 312, Mallory Hall

We begin with some general observations about partial orders on quotient spaces, and then explore their use in music theory, in two different contexts. In the first, we show that most of the familiar chord and scale types in Western music appear as extremal elements in a quotient of the partial order induced by set inclusion. In the second, we propose a partial order that models the "brightness" aspect of timbre, and use it to answer the following question: among all instruments that are no brighter than a trumpet, which has the timbre that is closest to an oboe?

Bob Sachs, George Mason University Half of All 2 x 2 Real Matrices Have Real Eigenvectors 3:15 PM, Room 412, Mallory Hall

Consider the set of all real 2 x 2 matrices. Each matrix generates a corresponding system of first-order differential equations. A simple diagram will illustrate that only those non-degenerate systems compatible with a symmetric matrix will have real eigenvectors. In this qualitative version a system is analyzed in terms of two lines in the plane (nullclines) and two choices of signs, hence four total types. Two of these are diagonalizable over the reals, hence the title.

Karin Saoub, Roanoke College Graph Theory as a General Education Course 3:15 PM, Room 313, Mallory Hall

Many liberal arts math courses include a chapter or unit on graph theory. In this talk, we explore ways to use graph theory as a basis for a semester long general education course. Applications include finding optimal delivery routes, shortest networks, and stable matchings.

Amy Shell-Gellasch, Hood College History the Hood Way

4:05 PM, Room 313, Mallory Hall

Many colleges and universities now offer a history of mathematics course, either for math majors, math education majors, or for general credit. At Hood College, we have made the history of mathematics a mainstay of majoring in mathematics. We weave the history of mathematics through all the courses in the major, culminating in our senior seminar. In this talk we will present the ways Hood College makes the history of mathematics an integral component of the math major.

David Shoenthal, Longwood University Arms Race Examples in Intro ODEs 3:15 PM, Room 413, Mallory Hall

We investigate an arms race question from a standard ODE text. While one usually sees a mixing problem with two tanks used to illustrate the topic of solving linear equations with constant coefficients, I will argue that a simple arms race problem leads quickly to many interesting questions.

Thomas Sibley, St. John's University and the College of St. Benedict

Puzzling Groups

11:05 AM, Room 312, Mallory Hall

Many puzzles involving turning pieces have associated permutation groups. A family of puzzles easy enough to explore and analyze in an abstract algebra class lead to a more general investigation.

Troy Siemers, Virginia Military Institute Becoming an Applied Mathematician 9:15 AM, Room 412, Mallory Hall

The VMI math department shifted its focus from pure mathematics to one with an applied emphasis. While I was trained in pure mathematics programs, I have since branched out in work at VMI and UVA with collaborators including a physicist, a biochemist, a psychologist, a biologist, engineers, and economists. These projects have included protein structure, laser start-up characteristics, immigration modeling, and most recently biofuel. This talk will include details on the projects and the mathematical skills involved.

Wendy Hageman Smith, Longwood University

How Does Applying A Constructivist Theory of Teaching Affect Student Outcomes and Attitudes in the Post-Secondary General Education Math Class?

8:50 AM, Room 313, Mallory Hall

General Education math students are notoriously math averse and math anxious. This makes achieving desired learning outcomes especially difficult for these students. My research at the University of Colorado indicated a significant positive change in outcomes for such students exposed to classroom methods and discourse informed by a constructivist theory of teaching. I now seek to replicate these results in a muli-institutional study encompassing diverse instructional styles and gen-ed math syllabi. The aim of the study is ultimately to provide instructors a framework for implementing best-practices in constructivist methodology in such courses. An outline of the proposed study will be presented, and the remainder of the talk will be devoted to open discussion of the best way to structure and implement the study.

Eve Torrence, Randolph-Macon College

Models of Icosahedra

9:15 AM, Room 413, Mallory Hall

We will examine the structure of a few of the 59 stellations of the icosahedron. Three dimensional models make these complex structures easier to understand. I will bring examples of the models that can be built from my book and I will explain how I created the nets for these polyhedra.

D. Brian Walton, James Madison University

Using WeBWorK Online Assignments to Build Student Understanding

11:05 AM, Room 412, Mallory Hall

WeBWorK is an open-source online homework system designed to provide a wide range of mathematically sophisticated problems. Over the last three years, I have been using WeBWorK as a learning tool in my calculus courses. I will provide an interactive demonstration of the system, including examples of problems that I have used along with a discussion of their pedagogical intent. Examples will include numeric and algebraic calculations, interpretation of graphs, and the step by-step application of limit rules.

Matthew Willis, Hampden-Sydney College Analyzing Sets of Demaxure Tableaux and their Convexity

9:15 AM, Room 312, Mallory Hall

Much like the more recognizable Schur functions, Demazure characters of finite-dimensional, irreducible L-modules in Case A can be produced using a tableau description. In this talk, we will study the set of tableaux that contribute to a particular Demazure character. We will also provide a necessary and sufficient condition for this set to form a convex polytope. If time allows, we will connect these Demazure characters to Flagged-Schur functions.

Student Abstracts by Author

Michael Mudarri (Senior), Hood College Constructive Proof of the Cubic Case of Kronecker-Weber 3:40 PM, Room 212, Mallory Hall

The Kronecker-Weber theorem, first proved at the end of the 19th century, states that any abelian extension of the rational numbers Q is contained in a cyclotomic extension of Q. Let f be a cubic polynomial with rational coefficients whose discriminant is a perfect square in Q. The Kronecker-Weber theorem implies that the roots of f can be expressed as cyclotomic numbers, i.e. as Q-linear combinations of roots of unity. The usual proofs of the theorem are not evidently constructive. I will discuss an algorithm for constructing a representation of the roots of f as cyclotomic numbers using the cubic formula and classical facts from the theory of cyclotomy.

Minh Nguyen (Sophomore), Liberty University

A Concurrency of Five Lines

4:05 PM, Room 213, Mallory Hall

Let *I* be the incenter of triangle *ABC*, and *O* be its circumcenter. Denote by A_1 , B_1 , and C_1 the points on *BC*, *CA*, and *AB*, respectively, such that IA_1 , IB_1 , and IC_1 in order are the symmedians of Triangle(*IBC*), Triangle(*ICA*), and Triangle(*IAB*). Let *Y* be the Gergonne point and *G* be the centroid of Triangle(*ABC*). In this talk, we discuss the concurrency of the five lines: AA_1 , BB_1 , CC_1 , OI, and GY.

April-Nicole Smith (Senior), Virginia State University Dustin Robinson (Senior), Virginia State University *Translates of Perfect Polygons and Geometric Triple Systems* 3:15 PM, Room 213, Mallory Hall

Let *P* be a set of *n* points in the plane, no three are collinear, labeled with Z_n . For each $k \in Z_n$, let W_k denote the set of lines, $W_k = \{(a, b) : a \neq b, a + b = k\}$. If for each *k*, the lines in W_k are concurrent, then *P* is a *perfect n-gon*. Let X_k denotes the part of concurrence of the lines in W_k . The set $\{X_k : k \in Z_n\}$ is called the *perspective set* of *P*. A *geometric triple system* $T(Z_n, k)$ is a set of *n* points in a plane, no four are collinear, labeled with Z_n such that every three element subset $\{a, b, c\}$ of Z_n with $a + b + c = k \pmod{n}$, corresponds to three collinear points. The vertices of a geometric triple system as well as the vertices and perspective points of the perfect polygon have been shown to lie on a *nonsingular irreducible cubic curve* α . A binary operation * on α can be defined by setting a * b to be the third point on the line (a, b) and on the curve α . If *q* is any point of α , then $q * p = \{q * k : k \in Z_n\}$ is called a translate of *P*. We show that every perfect polygon on α is a translate of a certain geometric triple system on α .

Mark Verdi (Junior), American University Properties of Polynomials with Closed Lill-paths 3:15 PM, Room 212, Mallory Hall

Lill's Theorem is a geometric method of finding roots of polynomials, in which a path is drawn whose side lengths correspond to the coefficients of each term of the polynomial. Following a suggestion of Francis Su, we consider polynomials whose Lill-paths are closed. We show that such polynomials always have roots at i and -i, and consider extensions to generalized closed paths.

Carson Wang (Junior), University of Virginia

Hope Gibbs (Junior), Virginia State University

Midsets of Perfect Polygons as Geometric Triple Systems

3:40 PM, Room 213, Mallory Hall

Let *P* be a set of *n* points in the plane, no three collinear, labeled with Z_n . For each $k \in Z_n$, let W_k denote the set of lines, $W_k = \{(a, b) : a \neq b, a + b = k\}$. If for each *k*, the lines in W_k are concurrent, then *P* is a *perfect n-gon*. A *geometric triple system* $T(Z_n, k)$ is a set of *n* points in the plane, no four collinear, which can be labeled with Z_n such that for every subset $\{a, b, c\}$ of Z_n with $a + b + c = k \pmod{n}$, the corresponding points *a*, *b*, and *c* are collinear. If *a* and *b* are any two vertices of a perfect polygon, then the *midset* S(a, b) generated by $\{a, b\}$ consists of the points $\{S_k : k \in Z_n\}$ where $S_k = (a, a+k) \circ (b, b-k)$. We show that every midset of a perfect polygon is a geometric triple system.